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# "PARAMETERIZATION OF THE BREMSSTRAHLUNG SPECTRUM"

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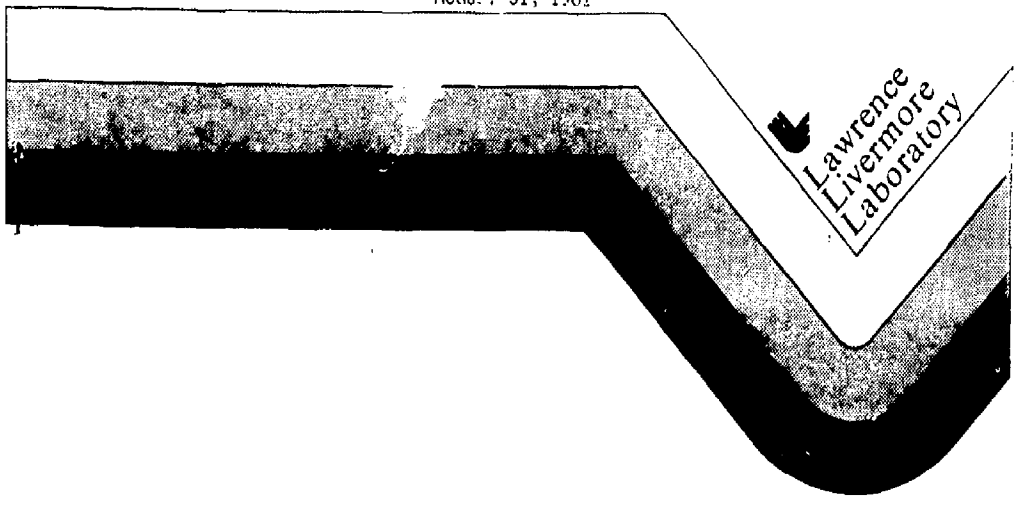




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# Parameterization of the Bremsstrahlung Spectrum

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## ABSTRACT

A parameterization is provided for the bremsstrahlung spectrum in the incident electron energy range 1 - 500 keV for all elements, with particular emphasis on the range 20 - 100 keV for Z = 41-92. A general accuracy of 20% is achieved, with 50% in the worst cases, in most ranges utilizing simple one variable functions.

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## I. Introduction

The objective of this work was to provide a parameterization of the bremsstrahlung spectrum and to compare its predictions with sample calculations. The focus was on the characterization of the spectrum for elements Z = 41-92 (but also considering lighter Z), for many degrees of ionization, for the range of incident electron kinetic energies 2 - 500 keV with particular emphasis on the range 20 - 100 keV. The ultimate objective was 5-10% accuracy, but the goal for this work was general 20% accuracy, 50% in worst cases.

These objectives have been generally achieved, and generally in terms of simple one variable functions. Our accuracy is generally 20% or better in the energy range 20 - 100 keV. By 500 keV 50% errors are found in the hard photon region of the spectrum from high Z elements. By 1 keV there are 50% errors in the soft photon region of the spectrum for both small Z and large Z elements.

Our strategy has been (1) to describe bremsstrahlung from ions in terms of bremsstrahlung from neutral atoms and bremsstrahlung from a point Coulomb charge, (2) to then also describe bremsstrahlung from neutral atoms in terms of bremsstrahlung from a point Coulomb charge, and (3) to show that simple characterizations are available for the various cases of the Coulomb spectrum. We therefore begin in Sect. II with a discussion of the point Coulomb case, followed by a discussion of screening in Sect. III and ions in Sect. IV. In Sect. V we identify problems which would need to be resolved to extend the range of validity of our approach or to improve its accuracy. In Sect. VI we summarize our current prescription for the parameterization of the bremsstrahlung spectrum and in Sect. VII we compare its predictions with numerical cal-

culations. Section VIII summarizes the conclusions which can be drawn about the parameterization of the bremsstrahlung spectrum.

## II. The Point Coulomb Case

Our treatment begins with the observation[1] that in the energy range of concern, for high Z elements nonrelativistic dipole approximation[2] is valid throughout (this is an illustration of the cancellation among relativistic, retardation and perhaps higher multipole effects which has been observed in a variety of processes), whereas for low Z elements once nonrelativistic dipole approximation fails relativistic Born approximation[3] combined with the Elwert factor[4] is good. The second observation is that the application of the nonrelativistic dipole approximation can be divided into a classical (Landau-Lifshitz[5]) regime and a quantum regime, and that when the classical result fails a nonrelativistic Born-Elwert result is adequate. The third observation is that the classical bremsstrahlung spectrum can be well characterized in terms of its two limiting forms: the soft photon limit (logarithmic divergence) and hard photon limit (Kramers formula[6]). The essential aspects of the regions of validity of these simple approximations are charted in Figs. 1-3, where Fig 1 shows the validity of simpler approximations relative to the Sommerfeld formula[2] and Fig. 2 (for Z=26) and Fig. 3 (for Z=74) show validity relative to exact numerical point Coulomb calculations.[1] These figures and approximations are discussed in subsections below, beginning with (a) the classical Landau-Lifshitz[5] formula and its characterization by hard and soft photon limits, (b) the Sommerfeld formula[2] and its characterization by either a classical result or nonrelativistic Born-Elwert, and (c) the full relativistic Coulomb case and the validity of relativistic

Born-Elwert (Bethe-Heitler-Elwert). The conclusion is that four simple formulas (three of them single-variable formulas) suffice.

a) Approximation for Classical Coulomb Bremsstrahlung

The classical bremsstrahlung radiation spectrum associated with motion of an electron (without energy loss) in the field of a point charge Z on a Coulomb trajectory (note the change of orbit with electron energy loss by radiation is neglected) is given by Landau and Lifshitz[5] (LL). One might suppose that the LL result would only be valid for radiation of photon energies k small compared to the incident electron kinetic energy $T_1$, but even for $k/T_1 \sim 1$ LL agrees with the semi-classical Kramers formula $\sigma(k) \simeq 5.61$ mb, and LL is indeed a valid approximation to the quantum Sommerfeld result for low $T_1$ even toward $k/T_1 \sim 1$. For $\sigma(k) \equiv \beta_1^4(k/Z^4)(d\sigma/dk)$ the result is

$$\sigma(k) = \frac{4\pi^2 k^3 \mu}{3} \; i\, H^{(1)}_{i\mu}(i\mu) \times H^{(1)\prime}_{i\mu}(i\mu) \qquad (1)$$

where $H^{(1)}$ and $H^{(1)\prime}$ are the Hankel function and its derivative respectively. The result is a one variable function of $\mu = \frac{\nu_1 k}{2T_1}$, (with $\nu_1 \equiv 2\alpha/\beta_1$, $\beta_1$ the incident electron velocity) which has been tabulated by Florescu and Costescu.[7] (See Table 1) The simple Kramers formula $\sigma = \frac{16\pi\alpha^3}{3\sqrt{3}} \simeq 5.61$ mb away from the soft photon end of the spectrum, together with $\sigma = (16\alpha^3/3)\ln(2/\gamma\mu)$, ($\gamma = e^C$, C = Euler's constant) at the soft photon end, may be combined to give a semiquantitative representation of the full classical result.

Further terms in the soft photon (smaller $\mu$) and the hard photon (large $\mu$)

expansions have been obtained by Florescu and Costescu.[7] The expression for small $\mu$ is

$$\sigma(k) = \frac{16\alpha^3}{3}(1 + \pi\mu)\ln\frac{2}{\gamma\mu} + O(\mu^2). \tag{2}$$

The expression for large $\mu$ is

$$\sigma(k) = \frac{16\pi\alpha^3}{3\sqrt{3}}[(1 + d_1\mu^{-2/3} + d_2\mu^{-8/3} + d_1d_2^{-2} + O(\mu^{-8/3})], \tag{3}$$

where $d_1 = 0.217747$, $d_2 = -0.0131214$, and $d_1d_2 = -1/350$. The comparison of LL with these more complete small $\mu$ and large $\mu$ expansions is given in tables 2-3. The switch over point between the two expressions is chosen to be $\mu \simeq 0.3$. With this choice the error of these approximations in comparison to the exact classical result is about 3% in the worst case.

b) Approximations for the Sommerfeld Formula

The electron bremsstrahlung spectrum which follows from the quantum theory of radiation in nonrelativistic dipole approximation was obtained by Sommerfeld.[2] The result is

$$\sigma(k) = \frac{16\pi^3}{3}\alpha^3 \frac{1}{(e^{2\pi\nu_1}-1)(1-e^{-2\pi\nu_2})} X_0$$

$$\times \frac{d}{dX_0}|F(i\nu_1, i\nu_2; 1; X_0)|^2, \tag{4}$$

with $\nu_i = Z\alpha/\beta_i$ and $X_0 = -4\nu_1\nu_2/(\nu_1-\nu_2)$, where F is the hypergeometric function.

A tabulation of the Sommerfeld formula is given by Florescu and Costescu[7] (table 4). They have also examined the circumstances in which this Sommerfeld formula begins to deviate from the classical prediction of LL, obtaining a delineation of the classical region similar to that shown in Fig. 1. For $\nu_1 \geq 5$ the difference between classical and nonrelativistic quantum predictions is less than 5% for almost the entire spectrum, while for higher energies (smaller $\nu_1$) classical results are increasingly restricted to the soft photon region of the spectrum (continuing to remain valid in predicting the existence and coefficient of the logarithmic divergence at the soft photon end of the spectrum).

For high energies (where $\nu_1$ and $\nu_2$ are both small) the Sommerfeld (S) formula reduces to the nonrelativistic Born approximation result

$$\sigma(\kappa) = \frac{16}{3}\alpha^3 \ell n \frac{p_1+p_2}{p_1-p_2}$$

away from the tip, while at the tip ($\nu_1$ small, $\nu_2$ large) it reduces to

$$\sigma = (16\pi/3)\alpha^3\nu_1.$$

These criteria are satisfied in a relatively restricted region where we shall see that in fact S is no longer valid (except for the very lightest elements) due to the need to include relativistic effects. Born approximation also gives the correct logarithmic (but not constant) term of the soft photon end point, for any choice of $\nu_1$ so that it remains valid sufficiently near this limit.

Born approximation can be improved[4] (Elwert-Born) by multiplying by the

factor $f_E = (\nu_2/\nu_1)\ [(1-e^{-2\pi\nu_1})/(1-e^{-2\pi\nu_2})]$ which can be identified as the ratio of the Sommerfeld prediction to Born approximation (note $f_E=1$ at the soft photon endpoint of the spectrum). This approximation was derived from S under condition that $2\pi(\nu_2-\nu_1)<<1$, but in fact it was found to have greater validity, extending to the tip region where $\nu_2$ becomes large. (At low energies EB predicts a tip value $\sigma \tilde{=} 6.2$ mb, not too different from Kramers, so that EB is good at both endpoints of spectrum for all $T_1$. However at low energy, in the classical region, EB gives a poor result throughout most of the spectrum.) The Elwert factor is correcting the Born normalization of the outgoing electron, and relaxes the requirement that $\nu_2$ be small, while largely leaving the requirement that $\nu_1$ be at least not too large.[8]

We show the validity of EB in Fig. 1. The important result is that its region of validity overlaps that of LL at the 10% level. If we use as a switching criteria $\nu_1[1-(k/T_1)] = 0.7$, we will not make errors of more than 13% (3% comes from the approximated classical result). In fact we may simplify EB to a one-variable formula by dropping the exponential factors in the Elwert factor. This will be valid if $2\pi Z\alpha/\beta_1 \geq 3$ or if $p_1 \sim p_2$. The bound of this region is also shown in Fig. 1. In the region beyond this bound, S is no longer valid (low Z) or we are beyond the energy range of interest (high Z). Hence for our purposes here we may take EB in its simplified one-variable form.

c) Approximations for the Full Relativistic Calculation

The validity of the Sommerfeld nonrelativistic dipole calculation is illustrated in Figs. 2 and 3. For high Z it may be used throughout our energy

range, while for low Z (Z=26) it fails at the 10% level for $v_1 < 0.5$.

Relativistic Born approximation (Bethe-Heitler[3]) is also available, but restricted in validity to low Z elements away from tip. In this approximation the spectrum becomes

$$\sigma(k) = \alpha^3 \frac{p_2}{p_1} \left\{ \frac{4}{3} - 2E_1E_2 \frac{p_1^2+p_2^2}{p_1^2p_2^2} + \frac{\varepsilon_1E_2}{p_1^3} + \frac{\varepsilon_2E_1}{p_2^3} - \frac{\varepsilon_1\varepsilon_2}{p_1p_2} + L\left[ \frac{8E_1E_2}{3p_1p_2} + \frac{k^2(E_1^3E_2^3+p_1^2p_2^2)}{p_1^3p_2^3} + \frac{k}{2p_1p_2}\left( \frac{E_1E_2+p_1^2}{p_1^3}\varepsilon_1 - \frac{E_1E_2+p_2^2}{p_2^3}\varepsilon_2 + \frac{2kE_1E_2}{p_1^2p_1^2} \right) \right] \right\},$$

where

$$L = 2\ell n\left( \frac{E_1E_2+p_1p_2-1}{k} \right), \qquad \varepsilon_1 = \ell n\left( \frac{E_1+p_1}{E_1-p_1} \right),$$

$$\varepsilon_2 = \ell n\left( \frac{E_2+p_2}{E_2-p_2} \right).$$

The exact Bethe Maximon high energy limit (away from the tip) indicates that BH makes errors of 10% in the limit.

The Elwert factor has been used to improve relativistic Born approximation (EBH), particularly at the tip end of the spectrum. For low Z elements this procedure gives satisfactory results for the entire spectrum.

We show in Fig. 2 and 3 the validity of EBH. Once again we find that, considering the overlap in the validity of EB and EBH, we can take a simple switching criteria $T_1 \simeq (2a)^4$, where $a = Z\alpha$.

A comparison of results obtained by this prescription with our point Coulomb data is given in Table 5. With these approximations, the overall error is less than 15% for Z=26, for energies $T_1$ = 5-200 keV, and 13% for Z = 74, for energies $T_1$ = 5-100 keV. For high Z (Z=74) and high energies ($T_1 \sim 500$ keV) this approximation gives a $\sim$50% error at the hard photon end.

## III. Screening

Screening of the nucleus by atomic electrons substantially reduces the bremsstrahlung cross sections, particularly at low energies. In the nonrelativistic quantum regime an accurate form factor approximation (EBF) gives a fairly good estimate of the reduction due to screening. In the relativistic regime of concern here screening effects are small, while in the classical regime, where Born approximation is no longer valid, EBF also fails. Comparisons of EBF and ES results are given in Tables 6-7. The ratio of EBF/ES to EB/EC pinpoints the error of Born approximation screening and indicates when other mechanisms become important. The ratio is close to 1 except for Z = 79 at 5 keV, as shown in Tables 5-6. The problem in this case is less serious at the tip, where EB has remained valid.

For our purposes here we shall demonstrate that we can choose an effective $Q_s^{ef}$ in a potential $(-Ze^2e^{-Q_s r})/r$, such that form factor approximation gives an adequate description of screening effects through the ranges of concern here. EBF has not reduced the cross section enough, which can be described as saying there is more screening than the form factor describes — i.e., $Q_s^{ef}$ is larger than $Q_s$. Comparisons of form factor results from various choices of $Q_s^{ef}$ and ES are given in Figs. 4-7 with $Q_s = Z^{1/3}/192$ (derived in

Jackson[10]), and the choice $Q_s^{ef} = \frac{3.0\ Z^{0.35}}{192}$, the EBF prediciton for the spectrum is

$$\sigma(k) = \frac{16}{3}\cdot\alpha^3\left[-\frac{Q_{max}^2}{2(Q_{min}^2+Q_s^{ef2})} + \frac{Q_{min}^2}{2(Q_{min}^2+Q_s^{ef2})} + \frac{1}{2}\,\ell n(Q_{max}^2+Q_s^{ef2}) - \frac{1}{2}\,\ell n(Q_{min}^2+Q_s^{ef2})\right] \tag{7}$$

We show in tables 8-10 how well this EBF reproduces ES data as well as how accurately it characterizes the screening effects.

## IV. Ions

The bremsstrahlung spectrum for ions may be obtained by interpolating between Coulomb ($\sigma_{EC}$) and "exact" screened ($\sigma_{ES}$) neutral atom results. Define[11] the "ionization factor"

$$I(k/T_1,\ T,\ Z,\ Z_i) = \frac{\sigma(k/T_1,T_1,Z,Z_i) - \sigma(k/T_1,T_1,Z,0)}{\sigma(k/T_1,T_1,Z,Z_i) - \sigma(k/T_1,T_1,Z,0)}\ .$$

Then the spectrum for ions can be written in terms of Coulomb and screened atom results as

$$\sigma(Z,Z_i) = I\sigma(Z,Z) - (I-1)\ \sigma(Z,0)\ , \tag{8}$$

where $\sigma(Z,Z)$ is the Coulomb spectrum and $\sigma(Z,0)$ is the screened neutral atom spectrum. The behavior of I was studied by Lee et. al.[11] with the results shown in Figs. 8-9. The ionization factor varies from 0 for neutral atoms to 1 for totally stripped atoms. When $k/T_1 = 0$, I is simply equal to $(Z_i/Z)^2$.

We choose, $I = (Z_1/Z)^3$, since except for low energies ($T_1 \leq 1$ keV) or very near the soft photon end, it gives a fairly good interpolation between Coulomb and screened results. Comparisons of results obtained by this prescription with our ionic data is given in table II. The error is no more than 15% except for Mo (Z=42) and W (Z=74) at $T_1 = 1$ keV.

## V. Remaining Problems

### 1. Coulomb case.

For high Z elements at high energies ($T_1 \sim 200$ keV) the EBH approximation gives too small a value in the tip region. We know that near the tip the p wave contributions of low energy outgoing electrons become important for high Z elements, and are responsible for EBH or EBF beign too small. Away from the tip the Bethe-Maximon results show that errors of 10% persist in BH. Another problem is to understand the switch from EB to EBH. We need more data for intermediate Z's (Z=42-79) to understand the underlying mechanism of the performance of EB and EBH.

### 2. Screening

At present we are using an effective form factor which remains adequate further into the classical screening region at the expense of a more correct description at higher energies where however screening effects become smaller.

To improve the screening predictions, we need to understand the behavior of the screened spectrum in the classical region. How to characterize screening in this region and how to switch to the quantum form factor approach remains to be established. Also we need to use a more adequate simple analytic characterization of form factors, together with an understanding of the sensitivity of

results to the features and detail of the form factor.

3. Ions.

A more sophisticated approximation to the ionization factor I may make it possible to achieve more satisfactory ionic data in the interpolating between Coulomb and neutral results. We will need additional ionic data to check the validity of various possible approaches.

4. Applications

These results must be used with caution for some applications, such as the calculation of the bremsstrahlung process in high temperature and high density plasmas. It is not necessarily realistic to use "isolated" ion or atom results by characterizing the $\bar{Z}_i$ for the plasma, or to use one average atom to characterize the constituents of the plasma. We need to compare results obtained from calculations within finite temperature average atom potentials (such as Thomas-Fermi) to establish the validity of the $\bar{Z}_i$ approach.

VI. Prescription

I. Coulomb case

A. $v_1(1 - \frac{k}{T_1}) < 0.7$ where ($v_1 = \frac{Z\alpha}{p_1}$, $p_1 = \sqrt{2T_1}$) use classical approximation.

1. $\mu \geq 0.3$ (where $\mu = \frac{v_1 k}{2T_1}$) use hard photon approximation (a).

2. $\mu < 0.3$ use soft photon approximation (b).

B. $\nu_1(1 - \frac{k}{T_1}) > 0.7$ use Elwert-Born

1. for $T_1 \leq (2a)^4$, where $a = Z\alpha^2$ use simplified nonrelativistic EB. (c)
2. for $T_1 > (2a)^4$, use Elwert-Bethe-Heitler. (d)

II. Screened case.

With the Yukawa form factor,

$$Q_s^{ef} = \frac{3.0\ Z^{0.35}}{192}$$

use screened Elwert-Born-form factor formula (e)

III. Ions

$$\sigma(Z,Z_i) = I\sigma(Z,Z) - (I-1)\sigma(Z,0)$$

where $I = (Z_i/Z)^3$.

Approximation formulas

a. $$\sigma(k) = \frac{16\pi a^3}{3\sqrt{3}} (1 + d_1\mu^{-2/3} + d_2\mu^{-4/3} + d_1d_2^{-2} + O(\mu^{-8/3}))$$

where $d_1 = 0.217747$, $d_2 = -0.0131214$ and $d_1d_2 = -\frac{1}{350}$.

b. $\sigma(k) = \frac{16\alpha^3}{3}(1 + \pi\mu)\ \ell n\ \frac{2}{\gamma\mu} + O(\mu^2)$

where $\gamma \simeq 1.78$.

c. $\sigma(k) = \frac{p_1}{p_2} \cdot \frac{16}{3}\alpha^3 \ell n \left(\frac{p_1 + p_2}{p_1 - p_2}\right)$,

where $p_1 = \sqrt{2T_1}$ and $p_2 = \sqrt{2T_1(1 - \frac{k}{T_1})}$ .

d. $\sigma(k) = \sigma_{BH}(k) \cdot f_E$,

where

$$\sigma_{BH}(k) = \alpha^3 \frac{p_2}{p_1}\left\{\frac{4}{3} - 2E_1E_2\frac{p_1^2 + p_2^2}{p_1^2p_2^2} + \frac{\varepsilon_1E_2}{p_1^3} + \frac{\varepsilon_2E_1}{p_2^3} - \frac{\varepsilon_1\varepsilon_2}{p_1p_2}\right.$$
$$+ L\left[\frac{8E_1E_2}{3p_1p_2} + \frac{k^2(E_1^2E_2^2 + p_1^2p_2^2)}{p_1^3p_2^3} + \frac{k}{2p_1p_2}\left(\frac{E_1E_2 + p_1^2}{p_1^3}\varepsilon_1\right.\right.$$
$$\left.\left.\left. - \frac{E_1E_2 + p_2^2}{p_2^3}\varepsilon_2 + \frac{2kE_1E_2}{p_2^2p_1^2}\right)\right]\right\},$$

$$L = 2\ell n\left(\frac{E_1E_2 + p_1p_2 - 1}{k}\right), \quad \varepsilon_1 = \ell n\left(\frac{E_1 + p_1}{E_1 - p_1}\right), \quad \varepsilon_2 = \ell n\left(\frac{E_2 + p_2}{E_2 - p_2}\right),$$

and $f_E = \frac{\nu_2}{\nu_1}\ \frac{1 - e^{-2\pi\nu_1}}{1 - e^{-2\pi\nu_2}}$, with $\nu_i = \frac{Z\alpha}{\beta_i}$. Note that relativistic kinematics are used in this formula.

$$e.\quad \sigma(k) = \frac{16}{3}\alpha^3 \left[ -\frac{Q_{max}^2}{2(Q_{min}^2+Q_s^{ef^2})} + \frac{Q_{min}^2}{2(Q_{min}^2+Q_s^{ef^2})} \right.$$

$$\left. + \frac{1}{2}\ln(Q_{max}^2 + Q_s^{ef^2}) - \frac{1}{2}\ln(Q_{min}^2+Q_s^{ef^2}) \right]$$

where $Q_{max} = p_1 + p_2$, $Q_{min} = p_1 - p_2$,

$$Q_s^{ef} = \frac{3.0 \times Z^{0.35}}{192} .$$

VII. Comparison of prescription with numerical data.

A. Point Coulomb

We show in table 5 the comparison of results obtained form our prescription with exact numerical point Coulomb data. The accuracy of our prescription is within 10% for $T_1 \sim 50$ keV, the error reaches 20% by $T_1 \sim 200$ keV in the tip region.

B. Neutral Atom

We show in table 8 the numerical data taken from our tabulation[12] for neutral atoms. In table 9 we show the results obtained from our prescription. In table 10 we show the results of their ratios. For low Z cases except near the soft photon end, our prescription gives a 15% accuracy for $T_1$ = 5-50 keV. For high Z cases the range is $T_1$ = 10-100 keV for the entire spectrum. At higher energies ($T_1$ > 200 keV), the prescription fails (becomes too large) in the hard photon end. At lower energies at the soft photon end ($T_1$ < 5 keV) the prescription remains adequate for intermediate Z, but is too small for low Z, too large for high Z.

C. *Ions*

Good agreement is obtained with our limited data for ions, generally within 10-20% except at 1 keV for intermediate and high Z cases.

## VIII. Conclusions

We have obtained a parameterization of the bremsstrahlung spectrum for calculating energy transport in laser produced plasmas. Accuracies achieved are 20% in general, 50% in the worst cases. A particularly striking result is the nearly complete characterization of the Coulomb case. Improvements in accuracy and range are possible, particularly for the neutral atom case. One also should be aware of the possible limitations in assuming a $\bar{Z}_i$ for a plasma and applying the isolated ion results obtained from our prescription.

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## Table Captions

Table 1. Tabulation of $\sigma_{cl}$ (taken from Ref. 7).

Table 2,3. Comparison of classical Landau Lifshitz result (eq. 1) with small $\mu$ and large $\mu$ expansions (eqs. 2-3).(taken from Ref. 7).

Table 4. Tabulation of the Sommerfeld formula from Florescu and Costescu.[7]

Table 5. Comparisons of approximated point Coulomb results with exact numerical results.

Table 6. Comparisons of EBF and ES and EBF/ES to EB/EC ratios for Z = 13.

Table 7. Same as above (Table 6) for Z = 79.

Table 8. EXACT numerical bremsstrahlung data for neutral atoms.

Table 9. Approximated bremsstrahlung data for neutral atoms obtained from Eq. (7).

Table 10. Ratio of approximated to exact numerical bremsstrahlung data (from Table 7 and Table 8).

Table 11. Comparisons for the ionic data.

## Figure Captions

Figure 1. Validity of nonrelativistic EB, LL with respect to Sommerfeld's formula.

Figure 2-3. Validity of nonrelativistic EB, S, LL and EBH with respect to exact numerical results (EC).

Figure 4-7. Comparison of EB-Yukawa form factor results with different choices of $Q_s^{ef}$ with ES.

Figure 8-9. $Z_i/Z$ dependence of ionization factor (Eq. 8).

Table 1. Tabulation of $\sigma_{c1}$.

| $\mu$ | $\sigma_{c1}$ | $\mu$ | $\sigma_{c1}$ |
|---|---|---|---|
| 0.01 | 15.028 | 0.12 | 9.2555 |
| 0.012 | 14.529 | 0.13 | 9.1172 |
| 0.014 | 14.114 | 0.14 | 8.9924 |
| 0.016 | 13.760 | 0.16 | 8.7751 |
| 0.018 | 13.452 | 0.18 | 8.5917 |
| 0.02 | 13.180 | 0.2 | 8.4341 |
| 0.022 | 12.937 | 0.22 | 8.2968 |
| 0.024 | 12.718 | 0.24 | 8.1758 |
| 0.026 | 12.519 | 0.26 | 8.0682 |
| 0.028 | 12.337 | 0.28 | 7.9716 |
| 0.03 | 12.169 | 0.3 | 7.8843 |
| 0.035 | 11.801 | 0.32 | 7.8049 |
| 0.04 | 11.490 | 0.34 | 7.7324 |
| 0 045 | 11.223 | 0.375 | 7.6192 |
| 0.05 | 10.988 | 0.4 | 7.5473 |
| 0.055 | 10.781 | 0.45 | 7.4214 |
| 0.06 | 10.596 | 0.5 | 7.3146 |
| 0.065 | 10.428 | 0.6 | 7.1422 |
| 0.07 | 10.276 | 0.7 | 7.0082 |
| 0.075 | 10.137 | 0.8 | 6.9005 |
| 0.08 | 10.009 | 0.9 | 6.8115 |
| 0.085 | 9.8907 | 1. | 6.7366 |
| 0.09 | 9.7810 | 1.25 | 6.5916 |
| 0.095 | 9.6789 | 1.5 | 6.4858 |
| 0.1 | 9.5834 | 1.75 | 6.4047 |
| | | 2. | 6.3401 |
| | | 2.5 | 6.2430 |

Table 2. Comparison of classical Landau Lifshitz result (eq. 1) with small $\mu$ and large $\mu$ expansions (eqs. 2–3).

| $\mu$ | $\sigma_{cl}$ | $\sigma_{cl}^{s,ph}$ |
|---|---|---|
| 0.0001 | 28.832 | 28.832 |
| 0.0005 | 23.886 | 23.886 |
| 0.001 | 21.774 | 21.775 |
| 0.005 | 16.987 | 16.996 |
| 0.0075 | 15.830 | 15.844 |
| 0.01 | 15.028 | 15.049 |
| 0.02 | 13.180 | 13.231 |
| 0.05 | 10.988 | 11.127 |
| 0.08 | 10.009 | 10.216 |
| 0.1 | 9.5834 | 9.8225 |
| 0.3 | 7.88 | 7.92 |

Table 3. Comparison of classical Landau Lifshitz result (eq. 1) with small $\mu$ and large $\mu$ expansions (eqs. 2-3).

| $\mu$ | $\sigma_{cl}$ | $\bar{\sigma}$ |
|---|---|---|
| 0.3 | 7.88 | 7.79 |
| 0.5 | 7.3146 | 7.2296 |
| 1. | 6.7366 | 6.7205 |
| 2. | 6.3401 | 6.3373 |
| 3. | 6.1728 | 6.1718 |
| 4. | 6.0768 | 6.0763 |
| 5. | 6.0134 | 6.0131 |
| 6. | 5.9677 | 5.9676 |
| 7. | 5.9331 | 5.9329 |
| 8. | 5.9057 | 5.9056 |
| 10. | 5.8648 | 5.8648 |
| 14. | 5.8133 | 5.8133 |
| 20. | 5.7698 | 5.7698 |
| $\infty$ | 5.6055 | 5.6055 |

Table 4. Tabulation of the Sommerfeld formula from Florescu and Costescu.[7]

| $k/T_1$ \ $\nu_1$ | 0.001 | 0.01 | 0.1 | 0.5 |
|---|---|---|---|---|
| 0.001 | 25.631 | 25.631 | 25.598 | 24.883 |
| 0.002 | 23.487 | 23.488 | 23.458 | 22.755 |
| 0.005 | 20.651 | 20.652 | 20.630 | 19.959 |
| 0.01 | 18.501 | 18.504 | 18.493 | 17.868 |
| 0.02 | 16.344 | 16.348 | 16.358 | 15.811 |
| 0.04 | 14.170 | 14.178 | 14.223 | 13.806 |
| 0.05 | 13.465 | 13.474 | 13.535 | 13.175 |
| 0.1 | 11.242 | 11.258 | 11.389 | 11.274 |
| 0.15 | 9.9040 | 9.9272 | 10.120 | 10.211 |
| 0.2 | 8.9264 | 8.9557 | 9.2066 | 9.4815 |
| 0.25 | 8.1440 | 8.1792 | 8.4858 | 8.9312 |
| 0.3 | 7.4832 | 7.5240 | 7.8850 | 8.4923 |
| 0.35 | 6.9041 | 6.9505 | 7.3656 | 8.1292 |
| 0.4 | 6.3829 | 6.4348 | 6.9043 | 7.8208 |
| 0.45 | 5.9035 | 5.9611 | 6.4858 | 7.5541 |
| 0.5 | 5.4548 | 5.5182 | 6.0996 | 7.3200 |
| 0.55 | 5.0281 | 5.0973 | 5.7379 | 7.1123 |
| 0.6 | 4.6163 | 4.6916 | 5.3946 | 6.9267 |
| 0.65 | 4.2130 | 4.2947 | 5.0647 | 6.7597 |
| 0.7 | 3.8119 | 3.9005 | 4.7437 | 6.6088 |
| 0.75 | 3.4059 | 3.5019 | 4.4274 | 6.4721 |
| 0.8 | 2.9859 | 3.0901 | 4.1114 | 6.3482 |
| 0.85 | 2.5382 | 2.6519 | 3.7910 | 6.2354 |
| 0.9 | 2.0377 | 2.1630 | 3.4610 | 6.1322 |
| 0.95 | 1.4214 | 1.5630 | 3.1219 | 6.0370 |
| 1. | | | 2.8847 | 5.9479 |

Table 4. Continued

| $k/T_1$ \ $\nu_1$ | 1. | 2. | 5. | 8. |
|---|---|---|---|---|
| 0.001 | 23.591 | 21.705 | 19.008 | 17.629 |
| 0.002 | 21.476 | 19.614 | 16.973 | 15.640 |
| 0.005 | 18.715 | 16.910 | 14.398 | 13.166 |
| 0.01 | 16.671 | 14.940 | 12.587 | 11.469 |
| 0.02 | 14.689 | 13.072 | 10.944 | 9.9747 |
| 0.04 | 12.801 | 11.350 | 9.5154 | 8.7221 |
| 0.05 | 12.219 | 10.833 | 9.1068 | 8.3734 |
| 0.1 | 10.511 | 9.3667 | 8.0045 | 7.4580 |
| 0.15 | 9.5921 | 8.611[illegible] | 7.4750 | 7.0326 |
| 0.2 | 8.9796 | 8.1267 | 7.1476 | 6.7747 |
| 0.25 | 8.5281 | 7.7768 | [illegible]194 | 6.5972 |
| 0.3 | 8.1748 | 7.5082 | 6.7485 | 6.4655 |
| 0.35 | 7.8871 | 7.2930 | 6.6141 | 6.3628 |
| 0.4 | 7.6460 | 7.1150 | 6.5047 | 6.2798 |
| 0.45 | 7.4396 | 6.9643 | 6.4134 | 6.2107 |
| 0.5 | 7.2600 | 6.8343 | 6.3356 | 6.1522 |
| 0.55 | 7.1014 | 6.7206 | 6.2681 | 6.1016 |
| 0.6 | 6.9598 | 6.6198 | 6.2089 | 6.0573 |
| 0.65 | 6.8324 | 6.5296 | 6.1563 | 6.0181 |
| 0.7 | 6.7166 | 6.4481 | 6.1091 | 5.9830 |
| 0.75 | 6.6107 | 6.3739 | 6.0664 | 5.9513 |
| 0.8 | 6.5133 | 6.3060 | 6.0276 | 5.9226 |
| 0.85 | 6.4233 | 6.2435 | 5.9920 | 5.8962 |
| 0.9 | 6.3395 | 6.1856 | 5.9592 | 5.8720 |
| 0.95 | 6.2614 | 6.1317 | 5.9288 | 5.8497 |
| 1. | 6.1883 | 6.0815 | 5.9005 | 5.8289 |

Table 4. Continued

| $k/T_1$ \ $\nu_1$ | 10. | 15. | 20. |
|---|---|---|---|
| 0.001 | 16.983 | 15.827 | 15.025 |
| 0.002 | 15.021 | 13.925 | 13.175 |
| 0.005 | 12.604 | 11.630 | 10.980 |
| 0.01 | 10.969 | 10.121 | 9.5687 |
| 0.02 | 9.5525 | 8.8530 | 8.4112 |
| 0.04 | 8.3865 | 7.8451 | 7.5132 |
| 0.05 | 8.0662 | 7.5748 | 7.2763 |
| 0.1 | 7.2357 | 6.8887 | 6.6832 |
| 0.15 | 6.8553 | 6.5819 | 6.4220 |
| 0.2 | 6.6266 | 6.3999 | 6.2681 |
| 0.25 | 6.4701 | 6.2763 | 6.1642 |
| 0.3 | 6.3544 | 6.1855 | 6.0881 |
| 0.35 | 6.2644 | 6.1152 | 6.0293 |
| 0.4 | 6.1918 | 6.0587 | 5.9821 |
| 0.45 | 6.1316 | 6.0119 | 5.9432 |
| 0.5 | 6.0806 | 5.9724 | 5.9103 |
| 0.55 | 6.0366 | 5.9384 | 5.8820 |
| 0.6 | 5.9981 | 5.9087 | 5.8574 |
| 0.65 | 5 9641 | 5.8825 | 5.8356 |
| 0.7 | 5.9337 | 5.8591 | 5.8162 |
| 0.75 | 5.9063 | 5.8380 | 5.7988 |
| 0.8 | 5.8814 | 5.8189 | 5.7829 |
| 0.85 | 5.8586 | 5.8014 | 5.7685 |
| 0.9 | 5.8377 | 5.7854 | 5.7552 |
| 0.95 | 5.8183 | 5.7706 | 5.7430 |
| 1. | 5.8004 | | |

Table 5. Comparison of results obtained from our prescription ($\sigma_{approx.}$) for the point Coulomb case with exact numerical point Coulomb data ($\sigma_{EC}$).

Point Coulomb

| $T_1$ | Z / $k/T_1$ | 13 $\sigma_{EC}$ | 13 $\sigma_{approx}$ | 13 $\frac{\sigma_{approx}}{\sigma_{EC}}$ | 26 $\sigma_{EC}$ | 26 $\sigma_{approx}$ | 26 $\frac{\sigma_{approx}}{\sigma_{EC}}$ | 42 $\sigma_{EC}$ | 42 $\sigma_{approx}$ | 42 $\frac{\sigma_{approx}}{\sigma_{EC}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 0.95 | | | | 6.252 | 6.282 | 1.004 | 6.1 | 6.282 | 1.029 |
| | 0.9 | 6.25 | 6.17 | .987 | | | | | | |
| | 0.8 | 6.45 | 6.42 | .995 | 6.476 | 6.645 | 1.026 | 6.3 | 6.645 | 1.055 |
| | 0.6 | 7.00 | 7.07 | 1.010 | 6.871 | 7.28 | 1.059 | 6.7 | 7.06 | 1.054 |
| | 0.5 | 7.36 | 7.50 | 1.019 | | | | | | |
| | 0.4 | 7.81 | 8.05 | 1.031 | | | | | | |
| | 0.2 | 9.36 | 9.84 | 1.051 | | | | | | |
| 10 | 0.99 | | | | | | | | | |
| | 0.95 | | | | 6.225 | 6.282 | 1.009 | 6.1 | 6.282 | 1.029 |
| | 0.9 | | | | | | | | | |
| | 0.8 | 6.208 | 6.09 | .981 | 6.502 | 6.645 | 1.022 | 6.3 | 6.645 | 1.055 |
| | 0.6 | | | | 6.984 | 7.280 | 1.042 | 6.8 | 7.28 | 1.071 |
| | 0.4 | | | | | | | | | |
| | 0.2 | | | | | | | | | |
| 50 | 0.99 | | | | | | | | | |
| | 0.95 | | | | 5.189 | 4.775 | .920 | 5.7 | 6.282 | 1.102 |
| | 0.9 | 4.17 | 4.03 | .966 | | | | | | |
| | 0.8 | 4.56 | 4.42 | .969 | 5.609 | 5.212 | .929 | 6.1 | 6.645 | 1.089 |
| | 0.6 | 5.55 | 5.39 | .971 | 6.354 | 5.998 | .944 | 6.9 | 7.28 | 1.055 |
| | 0.4 | 6.86 | 6.73 | .981 | | | | | | |
| | 0.2 | 9.08 | 8.92 | .982 | | | | | | |
| | 0.1 | 11.2 | 11.1 | .991 | | | | | | |
| 100 | 0.95 | 2.95 | 2.82 | .956 | 4.141 | 3.72 | .898 | 4.9 | 4.07 | .831 |
| | 0.9 | 3.16 | 3.03 | .959 | | | | | | |
| | 0.8 | | | | 4.649 | 4.22 | .907 | 5.5 | 4.57 | .831 |
| | 0.6 | | | | 5.570 | 5.13 | .921 | 6.3 | 5.42 | .860 |
| 200 | 0.95 | | | | 2.941 | 2.57 | .874 | | | |
| | 0.8 | | | | 3.563 | 3.15 | .884 | | | |
| | 0.6 | | | | 4.655 | 4.20 | .902 | | | |
| 500 | 0.99 | | | | | | | | | |
| | 0.95 | | | | 1.730 | 1.48 | .855 | | | |
| | 0.9 | 1.48 | 1.39 | .939 | | | | | | |
| | 0.8 | | | | 2.480 | 2.16 | .871 | | | |
| | 0.75 | 2.34 | 2.20 | .940 | | | | | | |
| | 0.6 | | | | 3.677 | 3.31 | .900 | | | |
| | 0.5 | 4.09 | 3.90 | .953 | | | | | | |
| | 0.4 | | | | | | | | | |
| | 0.2 | | | | | | | | | |

Table 5. Continued

| $T_1$ | Z / $k/T_1$ | 74 $\sigma_{EC}$ | 74 $\sigma_{approx}$ | 74 $\frac{\sigma_{approx}}{\sigma_{EC}}$ | 79 $\sigma_{EC}$ | 79 $\sigma_{approx}$ | 79 $\frac{\sigma_{approx}}{\sigma_{EC}}$ | 92 $\sigma_{EC}$ | 92 $\sigma_{approx}$ | 92 $\frac{\sigma_{approx}}{\sigma_{EC}}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 0.95 | 6.198 | 6.282 | 1.014 | | | | | | |
| | 0.9 | | | | | | | 6.19 | 6.40 | 1.034 |
| | 0.8 | 6.310 | 6.476 | 1.026 | 6.24 | 6.44 | 1.032 | 6.26 | 3.67 | .586 |
| | 0.6 | 6.504 | 6.645 | 1.022 | 6.46 | 6.60 | 1.022 | 6.43 | 6.52 | 1.014 |
| | 0.5 | | | | | | | | | |
| | 0.4 | | | | | | | | | |
| | 0.2 | | | | 7.50 | 7.48 | .997 | | | |
| 10 | 0.99 | | | | | | | | | |
| | 0.95 | 6.368 | 6.282 | .986 | | | | 6.35 | 6.28 | .989 |
| | 0.9 | | | | | | | | | |
| | 0.8 | 6.507 | 6.645 | 1.021 | | | | 6.47 | 6.65 | 1.028 |
| | 0.6 | 6.759 | 6.886 | 1.019 | | | | 6.69 | 6.73 | 1.006 |
| | 0.4 | | | | | | | 7.05 | 7.03 | .997 |
| | 0.2 | | | | | | | | | |
| 50 | 0.99 | | | | 6.45 | 6.20 | .961 | | | |
| | 0.95 | 6.424 | 6.282 | .978 | | | | 6.65 | 6.28 | .944 |
| | 0.9 | | | | | | | | | |
| | 0.8 | | | | 6.78 | 6.65 | .981 | 6.90 | 6.65 | .964 |
| | 0.6 | 7.217 | 7.278 | 1.008 | 7.26 | 7.28 | 1.003 | 7.32 | 7.28 | .994 |
| | 0.4 | | | | 7.95 | 8.21 | 1.033 | 7.94 | 7.78 | .980 |
| | 0.2 | | | | 9.30 | 9.38 | 1.009 | | | |
| | 0.1 | | | | 10.8 | 10.6 | .981 | | | |
| 100 | 0.95 | 5.914 | 6.282 | 1.062 | | | | 6.35 | 6.28 | .989 |
| | 0.9 | | | | | | | | | |
| | 0.8 | 6.337 | 6.645 | 1.049 | | | | 6.70 | 6.65 | .993 |
| | 0.6 | 7.025 | 7.280 | 1.036 | | | | 7.30 | 7.28 | .997 |
| 200 | 0.95 | 4.937 | 6.282 | 1.272 | | | | 5.54 | 6.28 | 1.134 |
| | 0.8 | 5.497 | 6.645 | 1.209 | | | | 6.03 | 6.65 | 1.103 |
| | 0.6 | 6.429 | 7.288 | 1.134 | | | | 6.85 | 7.28 | 1.063 |
| 500 | 0.99 | | | | 3.43 | 6.20 | .808 | | | |
| | 0.95 | 3.412 | 6.28 | 1.841 | | | | 4.07 | 6.28 | 1.543 |
| | 0.9 | | | | 3.83 | 6.40 | 1.671 | | | |
| | 0.8 | 4.092 | 6.64 | 1.623 | | | | 4.70 | 6.65 | 1.415 |
| | 0.75 | | | | | | | | | |
| | 0.6 | 5.168 | 7.28 | 1.409 | | | | 5.72 | 7.28 | 1.273 |
| | 0.5 | | | | 6.07 | 7.70 | 1.269 | | | |
| | 0.4 | | | | | | | 7.21 | 8.22 | 1.140 |
| | 0.2 | | | | | | | | | |

Table 6. Comparisons of EBF and ES and EBF/ES to EB/EC ratios for Z = 13.

| | | | $\beta_1^2 \frac{k}{Z^2} \frac{d\sigma}{dk}$ (Coul.) | | | $\beta_1^2 \frac{k}{Z^2} \frac{d\sigma}{dk}$ (Neutral) | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | EB | | EC | EBF | | ES | EBF/ES |
| $\nu_1$ | $T_1$ (keV) | $k/T_1$ | mb | EB/EC | mb | mb | EBF/ES | mb | EB/EC |
| .68 | 5 | 0.9 | 6.17 | 0.987 | 6.25 | 5.37 | 0.963 | 5.58 | 0.976 |
| | | 0.8 | 6.43 | 0.996 | 6.45 | 5.54 | 0.971 | 5.70 | 0.975 |
| | | 0.6 | 7.08 | 1.01 | 7.00 | 5.93 | 0.991 | 5.98 | 0.981 |
| | | 0.5 | 7.51 | 1.02 | 7.36 | 6.12 | 0.999 | 6.13 | 0.979 |
| | | 0.4 | 8.05 | 1.03 | 7.81 | 6.29 | 1.004 | 6.27 | 0.975 |
| | | 0.2 | 9.84 | 1.05 | 9.36 | 6.43 | 0.989 | 6.50 | 0.942 |
| | | 0.0 | ∞ | 1 | ∞ | 6.28 | 0.974 | 6.63 | 0.974 |
| .23 | 50 | 0.9 | 4.03 | 0 967 | 4.17 | 3.98 | 0.961 | 4.14 | 0.994 |
| | | 0.8 | 4.42 | 0.969 | 4.56 | 4.34 | 0.969 | 4.48 | 1.000 |
| | | 0.6 | 5.40 | 0.973 | 5.55 | 5.22 | 0.963 | 5.42 | 0.990 |
| | | 0.4 | 6.73 | 0.981 | 6.86 | 6.34 | 0.962 | 6.59 | 0.981 |
| | | 0.2 | 8.93 | 0.984 | 9.08 | 7.94 | 0.958 | 8.29 | 0.974 |
| | | 0.1 | 11.0 | 0.982 | 11.2 | 8.92 | 0.952 | 9.37 | 0.970 |
| | | 0.0 | ∞ | 1 | ∞ | 9.60 | 0.932 | 10.3 | 0.932 |
| .068 | 500 | 0.9 | 1.39 | 0.938 | 1.48 | 1.39 | 0.944 | 1.47 | 1.060 |
| | | 0.75 | 2.20 | 0.941 | 2.34 | 2.19 | 0.943 | 2.33 | 1.002 |
| | | 0.5 | 3.90 | 0.954 | 4.09 | 3.85 | 0.951 | 4.05 | 0.997 |
| | | 0.0 | ∞ | 1 | ∞ | 10.9 | 0.916 | 11.9 | 0.916 |

Table 7. Comparisons of EBF and ES and EBF/ES to EB/EC ratios for Z = 79.

| | | $\beta_1^2 \frac{k}{Z^2} \frac{d\sigma}{dk}$ (Coul). | | | $\beta_1^2 \frac{k}{Z^2} \frac{d\sigma}{dk}$ (Neutral) | | | |
|---|---|---|---|---|---|---|---|---|
| $T_1$ | | EB | | EC | EBF | | ES | EBF/ES |
| (keV) | $k/T_1$ | mb | EB/EC | mb | mb | EBF/ES | mb | EB/EC |
| 5 | 0.99 | | | | 4.56 | 1.35 | 3.38 | |
| | 0.9 | | | | 4.64 | 1.37 | 3.40 | |
| | 0.8 | 6.52 | 1.04 | 6.24 | 4.73 | 1.43 | 3.31 | 1.38 |
| | 0.6 | 7.17 | 1.15 | 6.46 | 4.87 | 1.56 | 3.11 | 1.36 |
| | 0.4 | | | | 4.92 | 1.73 | 2.85 | |
| | 0.2 | 9.90 | 1.32 | 7.50 | 4.84 | 1.95 | 2.48 | 1.48 |
| | 0.1 | | | | 4.73 | 2.05 | 2.31 | |
| | 0.0 | ∞ | 1 | ∞ | 4.57 | 2.05 | 2.23 | 2.05 |
| 50 | 0.99 | 4.95 | 0.767 | 6.45 | 4.63 | 0.828 | 5 59 | 1.08 |
| | 0.9 | | | | 4.83 | 0.852 | 5.67 | |
| | 0.8 | 5.51 | 0.813 | 6.78 | 5.07 | 0.876 | 5.79 | 1.08 |
| | 0.6 | 6.27 | 0.864 | 7.26 | 5.64 | 0.931 | 6.06 | 1.08 |
| | 0.4 | 7.37 | 0.927 | 7.95 | 6 32 | 0 995 | 6.35 | 1.07 |
| | 0.2 | 9.31 | 1.001 | 9.30 | 7.17 | 1.05 | 6 80 | 1.05 |
| | 0.1 | 11.3 | 1.05 | 10.8 | 7.57 | 1.08 | 6.98 | 1.03 |
| | 0.05 | 13.1 | 1.07 | 12.3 | | | | |
| | 0.0 | ∞ | 1 | ∞ | 7.66 | 1.06 | 7.18 | 1.06 |
| 500 | 0.99 | 1.77 | 0.515 | 3.43 | 1.73 | 0.535 | 3.23 | 1.04 |
| | 0.9 | 2.10 | 0.548 | 3.83 | 2.05 | 0.565 | 3.62 | 1.03 |
| | 0.5 | 4.29 | 0.706 | 6.07 | 4.06 | 0.718 | 5.66 | 1.02 |
| | 0.0 | ∞ | 1 | ∞ | 9.67 | 0.843 | 11.47 | 0.843 |

**Table 8. EXACT numerical bremsstrahlung data for neutral atoms.**

Z= 13

| K/T1 / T1 | 0.000 | 0.100 | 0.200 | 0.300 | 0.400 | 0.500 | 0.600 | 0.700 | 0.800 | 0.900 | 0.950 | 0.990 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 3.646 | 3.773 | 3.873 | 3.948 | 3.998 | 4.027 | 4.043 | 4.048 | 4.049 | 4.050 | 4.050 | 4.050 |
| 2.5 | 5.234 | 5.338 | 5.439 | 5.527 | 5.593 | 5.636 | 5.661 | 5.674 | 5.680 | 5.682 | 5.682 | 5.682 |
| 5.0 | 6.557 | 6.694 | 6.831 | 6.957 | 7.054 | 7.114 | 7.147 | 7.164 | 7.172 | 7.175 | 7.175 | 7.175 |
| 10.0 | 7.777 | 7.924 | 7.576 | 7.234 | 6.935 | 6.705 | 6.563 | 6.487 | 6.450 | 6.435 | 6.433 | 6.433 |
| 25.0 | 9.291 | 8.942 | 8.227 | 7.537 | 6.950 | 6.495 | 6.230 | 6.093 | 6.026 | 6.000 | 5.996 | 5.995 |
| 50.0 | 10.303 | 9.461 | 8.348 | 7.418 | 6.654 | 6.091 | 5.749 | 5.575 | 5.490 | 5.458 | 5.453 | 5.452 |
| 100.0 | 11.028 | 9.575 | 8.130 | 7.119 | 6.334 | 5.799 | 4.742 | 4.568 | 3.644 | 3.169 | 2.956 | 2.915 |
| 200.0 | 11.504 | 9.372 | 7.687 | 6.448 | 5.479 | 4.671 | 3.975 | 3.555 | 3.349 | 3.253 | 1.906 | 1.865 |
| 500.0 | 12.066 | 9.174 | 7.266 | 5.916 | 4.856 | 2.988 | 3.259 | 2.682 | 2.542 | 1.474 | 1.436 | .389 |

Z= 26

| K/T1 / T1 | 0.000 | 0.100 | 0.200 | 0.300 | 0.400 | 0.500 | 0.600 | 0.700 | 0.800 | 0.900 | 0.950 | 0.990 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 2.188 | 2.295 | 2.389 | 2.469 | 2.536 | 2.587 | 2.722 | 2.841 | 2.929 | 2.985 | 3.004 | 3.009 |
| 2.5 | 3.567 | 3.715 | 3.797 | 3.854 | 3.895 | 3.918 | 3.961 | 4.000 | 4.028 | 4.045 | 4.051 | 4.053 |
| 5.0 | 4.917 | 5.072 | 5.168 | 5.230 | 5.245 | 4.996 | 4.952 | 4.922 | 4.906 | 4.900 | 4.899 | 4.899 |
| 10.0 | 6.268 | 6.462 | 6.427 | 6.313 | 6.149 | 5.914 | 5.831 | 5.802 | 5.792 | 5.789 | 5.789 | 5.789 |
| 25.0 | 7.984 | 7.956 | 7.606 | 7.244 | 6.874 | 6.592 | 6.324 | 6.152 | 6.057 | 6.020 | 6.014 | 6.013 |
| 50.0 | 9.226 | 8.817 | 8.085 | 7.427 | 6.875 | 6.419 | 6.033 | 5.716 | 5.533 | 5.459 | 5.447 | 5.446 |
| 100.0 | 10.253 | 9.291 | 8.163 | 7.252 | 6.519 | 5.925 | 5.383 | 4.933 | 4.595 | 4.427 | 4.365 | 4.356 |
| 200.0 | 11.039 | 9.365 | 7.902 | 6.785 | 5.905 | 5.153 | 4.533 | 3.985 | 3.571 | 3.384 | 3.317 | 3.304 |
| 500.0 | 11.676 | 9.269 | 7.494 | 6.196 | 5.174 | 4.315 | 3.617 | 3.098 | 2.733 | 2.588 | 2.537 | 2.528 |

Z= 42

| K/T1 / T1 | 0.000 | 0.100 | 0.200 | 0.300 | 0.400 | 0.500 | 0.600 | 0.700 | 0.800 | 0.900 | 0.950 | 0.990 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.357 | 1.452 | 1.552 | 1.649 | 1.742 | 1.830 | 1.914 | 1.995 | 2.077 | 2.133 | 2.157 | 2.163 |
| 2.5 | 2.456 | 2.603 | 2.732 | 2.843 | 2.935 | 3.002 | 3.044 | 3.069 | 3.083 | 3.088 | 3.090 | 3.090 |
| 5.0 | 3.685 | 3.864 | 3.988 | 4.059 | 4.111 | 4.151 | 4.185 | 4.221 | 4.250 | 4.270 | 4.278 | 4.281 |
| 10.0 | 5.258 | 5.479 | 5.543 | 5.547 | 5.511 | 5.462 | 5.428 | 5.401 | 5.385 | 5.378 | 5.376 | 5.376 |
| 25.0 | 6.941 | 7.053 | 6.895 | 6.692 | 6.471 | 6.275 | 6.121 | 5.993 | 5.913 | 5.881 | 5.874 | 5.873 |
| 50.0 | 8.330 | 8.207 | 7.723 | 7.247 | 6.838 | 6.497 | 6.226 | 5.998 | 5.859 | 5.807 | 5.797 | 5.795 |
| 100.0 | 9.716 | 9.142 | 8.152 | 7.411 | 6.786 | 6.273 | 5.837 | 5.465 | 5.193 | 4.984 | 4.911 | 4.896 |
| 200.0 | 10.750 | 9.416 | 8.139 | 7.137 | 6.331 | 5.682 | 5.201 | 4.697 | 4.314 | 3.959 | 3.821 | 3.792 |
| 500.0 | 11.516 | 9.416 | 7.774 | 6.54 | 5.565 | 4.769 | 4.157 | 3.649 | 3.311 | 2.934 | 2.784 | 2.757 |

Table 8. Continued

Z= 54

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | 0.975 | 1.051 | 1.133 | 1.219 | 1.306 | 1.399 | 1.499 | 1.603 | 1.697 | 1.751 | 1.756 | 1.774 |
| 2.5 | 1.901 | 2.039 | 2.171 | 2.292 | 2.409 | 2.524 | 2.638 | 2.748 | 2.836 | [illegible] | [illegible] | [illegible] |
| 5.0 | 3.645 | 3.236 | 3.307 | 3.[illegible]6 | 3.617 | 3.693 | 3.768 | 3.834 | 3.881 | 3.941 | [illegible] | [illegible] |
| 10.0 | 4.417 | 4.628 | 4.736 | 4.789 | 4.812 | 4.819 | 4.821 | 4.825 | 4.830 | 4.836 | [illegible] | [illegible] |
| 25.0 | 6.348 | 6.514 | 6.429 | 6.[illegible]9 | 6.139 | 5.979 | 5.843 | 5.731 | 5.648 | 5.[illegible]74 | [illegible] | [illegible] |
| 50.0 | 7.938 | 7.919 | 7.445 | 7.[illegible]62 | 6.772 | 6.434 | 6.[illegible]05 | 5.975 | 5.[illegible]73 | 5.[illegible] | [illegible] | [illegible] |
| 100.0 | 9.39[illegible] | 8.868 | 8.[illegible]05 | 7.[illegible]39 | 6.889 | 6.[illegible]36 | 6.[illegible]35 | [illegible].699 | [illegible].414 | 5.[illegible]43 | [illegible] | [illegible] |
| 200.0 | 10.592 | 9.443 | 8.271 | 7.[illegible]38 | 6.[illegible]67 | 5.972 | 5.436 | 4.[illegible]68 | 4.555 | 4.[illegible]94 | [illegible] | [illegible] |
| 500.0 | 11.492 | 9.[illegible]35 | 7.[illegible]75 | 6.787 | [illegible].838 | 5.[illegible]71 | 4.376 | 3.779 | 3.[illegible] | [illegible].782 | [illegible] | [illegible] |

Z= 74

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | [illegible].614 | 2.665 | [illegible].723 | [illegible].787 | [illegible].858 | [illegible].[illegible]19 | 1.[illegible]35 | 1.146 | 1.[illegible]55 | 2.[illegible]68 | 1.[illegible] | 1.[illegible]54 |
| 2.5 | 1.325 | 1.437 | 1.552 | 1.667 | 1.776 | 1.[illegible]71 | 2.[illegible]49 | [illegible].[illegible]13 | 2.[illegible]80 | 2.[illegible]44 | [illegible].[illegible] | 2.[illegible]56 |
| 5.0 | 2.336 | 2.[illegible]55 | 2.677 | 2.[illegible]22 | 2.[illegible]56 | 3.[illegible]92 | 3.196 | 3.[illegible]90 | 3.[illegible]76 | 3.[illegible]62 | 3.[illegible]54 | 3.[illegible]79 |
| 10.0 | 3.61[illegible] | 3.[illegible]29 | 3.[illegible]73 | 4.[illegible]74 | 4.151 | 4.[illegible]12 | 4.[illegible]7[illegible] | 4.[illegible]27 | 4.[illegible]29 | 4.[illegible]62 | 4.[illegible] | 4.[illegible]47 |
| 25.0 | 5.563 | 5.[illegible]6[illegible] | 5.[illegible]63 | 5.[illegible]7[illegible] | 5.616 | 5.[illegible]38 | 5.[illegible]89 | 5.[illegible]25 | 5.[illegible]90 | [illegible].4[illegible] | [illegible].[illegible]13 | [illegible].[illegible]45 |
| 50.0 | 7.226 | 7.[illegible]63 | 7.[illegible]18 | 6.[illegible]68 | 6.[illegible]97 | 6.[illegible]58 | 6.[illegible]71 | [illegible].[illegible]13 | [illegible].[illegible] | [illegible].[illegible]73 | [illegible].[illegible] | [illegible].[illegible]7 |
| 100.0 | 8.935 | 8.[illegible]92 | 7.[illegible]79 | 7.[illegible]26 | 6.[illegible]61 | 6.[illegible]59 | 6.237 | [illegible].[illegible]53 | [illegible].[illegible]14 | [illegible].[illegible]18 | [illegible].[illegible] | [illegible].[illegible]1 |
| 200.0 | 10.355 | 9.[illegible]31 | 8.[illegible]13 | 7.[illegible]6[illegible]7 | 6.[illegible]42 | 6.[illegible]3[illegible]2 | 5.[illegible]12 | [illegible].[illegible]70 | [illegible].[illegible]1[illegible] | [illegible].[illegible]71 | [illegible].[illegible] | [illegible].[illegible]1 |
| 500.0 | 11.413 | 9.[illegible]13 | 8.[illegible]78 | 7.[illegible]67 | 6.[illegible]71 | 5.[illegible]58 | 4.[illegible]91 | [illegible].[illegible]25 | [illegible].[illegible]14 | [illegible].[illegible]77 | [illegible].[illegible] | [illegible].[illegible]1 |

Z= 82

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | [illegible].534 | [illegible].582 | [illegible].636 | [illegible].695 | [illegible].760 | [illegible].[illegible]33 | [illegible].916 | 1.[illegible]11 | 1.[illegible] | [illegible].[illegible]37 | 1.[illegible]54 | [illegible].[illegible]4 |
| 2.5 | 1.186 | 1.29[illegible] | 1.40[illegible] | 1.5[illegible]9 | 1.622 | 1.740 | 1.865 | 1.[illegible]92 | [illegible].[illegible]1[illegible] | [illegible].[illegible]5[illegible] | [illegible].[illegible] | [illegible].[illegible]3 |
| 5.0 | 2.143 | 2.[illegible]54 | 2.471 | 2.612 | 2.746 | 2.873 | 2.991 | [illegible].[illegible]99 | [illegible].[illegible]19 | [illegible].[illegible]25 | [illegible].[illegible]4 | [illegible].[illegible]34 |
| 10.0 | 3.361 | 3.[illegible]76 | 3.725 | 3.[illegible]96 | 3.925 | 4.[illegible]0[illegible] | 4.[illegible]77 | 4.[illegible]39 | 4.[illegible]19 | 4.[illegible]24 | 4.[illegible] | 4.[illegible]3 |
| 25.0 | 5.[illegible]4 | 5.[illegible]29 | 5.529 | 5.498 | 5.424 | 5.[illegible]59 | 5.[illegible]31 | 5.[illegible]95 | 5.[illegible]1 | [illegible].[illegible]94 | [illegible].[illegible]11 | [illegible].[illegible]33 |
| 50.0 | 6.998 | 7.[illegible]64 | 6.859 | 6.[illegible]12 | 6.[illegible]3[illegible] | 6.[illegible]79 | 6.[illegible]1[illegible] | 5.[illegible]49 | 5.[illegible]7[illegible] | 5.[illegible]69 | 5.[illegible]29 | [illegible].[illegible]54 |
| 100.0 | 8.779 | 8.[illegible]82 | 7.[illegible]92 | 7.[illegible]4[illegible] | 6.971 | 6.[illegible]6[illegible]4 | 6.[illegible]34 | [illegible].[illegible]2[illegible] | [illegible].[illegible]1[illegible] | [illegible].[illegible]1[illegible] | [illegible].[illegible]9 | [illegible].[illegible]1 |
| 200.0 | 10.254 | 9.[illegible]0[illegible] | 8.443 | 7.[illegible]67[illegible] | 7.[illegible]33 | 6.[illegible]09 | 6.[illegible]42 | [illegible].[illegible]44 | [illegible].[illegible]0[illegible] | 4.[illegible]8[illegible] | [illegible].[illegible]4 | [illegible].[illegible]7 |
| 500.0 | 11.386 | 9.769 | 8.357 | 7.[illegible]3[illegible]5 | 6.432 | 5.[illegible]7[illegible]7 | [illegible].[illegible]3[illegible] | 4.[illegible]49 | [illegible].[illegible]7 | [illegible].[illegible]3[illegible] | [illegible].[illegible]3[illegible] | [illegible].[illegible]1[illegible] |

Table 8. Continued

Z= 92

| R/T1 T1 | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 0.462 | 0.507 | 0.559 | 0.615 | 0.676 | .737 | 0.8 2 | 0.869 | 0.936 | 1.0 2 | 1.0 1 | 1.0 1 | |
| 2.5 | 1.056 | 1.153 | 1.256 | 1.359 | 1.464 | 1.570 | 1.676 | 1.781 | 1.883 | 1.976 | 2.0 3 | 2.0 3 | |
| 5.0 | 1.951 | 2.114 | 2.257 | 2.389 | 2.516 | 2.638 | 2.755 | 2.867 | 2.977 | 3.0 8 | 3.143 | 3.0 4 | |
| 10.0 | 3.588 | 3.897 | 3.45 | 3.568 | 3.668 | 3.765 | 3.857 | 3.945 | 4.027 | 4.1 1 | 4.133 | 4.184 | |
| 25.0 | 5.0 7 | 5.419 | 5.262 | 5.242 | 5.199 | 5.138 | 5.134 | 5.122 | 5.121 | 5.142 | 5.157 | 5.1 4 | |
| 50.0 | 6.74 | 6.935 | 6.672 | 6.461 | 6.256 | 6. 77 | 5.9 0 | 5.8 6 | 5.711 | 5.644 | 5.6 0 | 5.6 | |
| 100.0 | 8.584 | 8.343 | 7.845 | 7.374 | 6.976 | 6.64 | 6.3 9 | 6.119 | 5.937 | 5.7 | 5.6 | 5.55 | |
| 200.0 | 10.11 | 9.351 | 8.452 | 7.78 | 7.114 | 6.6 | 6.187 | 5.9 7 | 5.813 | 5.2 1 | 5.1 | 4.97 | |
| 500.0 | 11.369 | 9.233 | 8.431 | 7.47 | 6.625 | 5.9 4 | 5.5 9 | 4.1 4 | 4.9 | 4. 7 | 3. | 3.742 | |

Table 9. Approximated bremsstrahlung data for neutral atoms obtained from Eq. (7).

Z= 13

| K/T1 / T1 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 2.379 | 2.437 | 2.504 | 2.581 | 2.666 | 2.760 | 2.861 | 2.964 | 3.[illegible] | 3.[illegible] | 3.[illegible] | [illegible] |
| 2.5 | 3.636 | 3.751 | 3.882 | 4.[illegible] | 4.17[illegible] | 4.3[illegible] | 4.422 | 4.517 | 4.[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |
| 5.0 | 4.650 | 4.[illegible] | 4.975 | 5.137 | 5.267 | 5.3[illegible] | 5.3[illegible] | 5.296 | 5.37[illegible] | 5.3[illegible] | 5.[illegible] | [illegible] |
| 10.0 | 5.692 | 5.868 | 6.[illegible] | 6.13[illegible] | 6.137 | 6.[illegible] | 5.963 | 5.934 | 5.7[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 25.0 | 7.039 | 7.245 | 7.[illegible] | 7.[illegible] | 6.77[illegible] | 6.4[illegible] | 6.[illegible] | 5.778 | 5.4[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 50.0 | 8.154 | 8.241 | 7.979 | 7.469 | 6.916 | 5.[illegible] | 5.[illegible] | 5.464 | 5.[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |
| 100.0 | 9.221 | 9.153 | 8.47[illegible] | 7.649 | 6.9[illegible] | 6.[illegible] | 5.646 | 5.[illegible] | 4.[illegible] | 4.[illegible] | 3.[illegible] | [illegible] |
| 200.0 | 10.295 | 9.917 | 8.756 | 7.[illegible] | 6.822 | 6.[illegible] | 5.[illegible] | 4.771 | 4.163 | 3.[illegible] | 3.[illegible] | [illegible] |
| 500.0 | 11.704 | 10.614 | 8.923 | 7.677 | 6.698 | 5.[illegible] | 5.134 | 4.439 | 3.745 | 2.[illegible] | 2.[illegible] | [illegible] |

Z= 26

| K/T1 / T1 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.783 | 1.816 | 1.854 | 1.899 | 1.948 | 2.[illegible] | 2.[illegible] | 2.141 | 2.[illegible] | 2.293 | 2.[illegible] | [illegible] |
| 2.5 | 2.954 | 3.039 | 3.136 | 3.245 | 3.363 | 3.4[illegible] | 3.[illegible] | 3.722 | 3.[illegible] | 3.[illegible] | 3.[illegible] | [illegible] |
| 5.0 | 3.936 | 4.[illegible] | 4.218 | 4.375 | 4.527 | 4.6[illegible] | 4.765 | 4.[illegible] | 4.[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |
| 10.0 | 4.941 | 5.142 | 5.335 | 5.5[illegible] | 5.630 | 5.[illegible] | 5.[illegible] | 5.[illegible] | 5.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 25.0 | 6.348 | 6.571 | 6.749 | 6.796 | 6.726 | 6.[illegible] | 6.416 | 6.[illegible] | 6.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 50.0 | 7.478 | 7.621 | 7.65[illegible] | 7.449 | 7.145 | 6.[illegible] | 6.515 | 6.[illegible] | 5.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 100.0 | 8.474 | 8.607 | 8.317 | 7.[illegible] | 7.254 | 6.767 | 6.336 | 5.[illegible] | 5.[illegible] | 5.[illegible] | 4.[illegible] | [illegible] |
| 200.0 | 9.542 | 9.481 | 8.731 | 7.9[illegible] | 7.182 | 6.[illegible] | 6.[illegible] | 5.551 | 5.[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |
| 500.0 | 10.955 | 10.357 | 8.98[illegible] | 7.872 | 6.992 | 6.255 | 5.6[illegible] | 5.[illegible] | 4.[illegible] | 3.[illegible] | 3.[illegible] | [illegible] |

Z= 42

| K/T1 / T1 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.411 | 1.431 | 1.453 | 1.478 | 1.509 | 1.543 | 1.583 | 1.629 | 1.68[illegible] | 1.734 | 1.796 | 1.[illegible] |
| 2.5 | 2.513 | 2.567 | 2.641 | 2.724 | 2.817 | 2.9[illegible] | 3.[illegible] | 3.[illegible] | 3.[illegible] | 3.[illegible] | 3.[illegible] | [illegible] |
| 5.0 | 3.453 | 3.562 | 3.6[illegible] | 3.822 | 3.962 | 4.[illegible] | 4.221 | 4.[illegible] | 4.4[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |
| 10.0 | 4.461 | 4.621 | 4.795 | 4.968 | 5.117 | 5.2[illegible] | 5.291 | 5.[illegible] | 5.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 25.0 | 5.837 | 6.[illegible] | 6.272 | 6.[illegible] | 6.451 | 6.[illegible] | 6.[illegible] | 6.[illegible] | 6.[illegible] | 6.[illegible] | 6.[illegible] | [illegible] |
| 50.0 | 6.894 | 7.147 | 7.293 | 7.263 | 7.1[illegible] | 6.[illegible] | 6.666 | 6.[illegible] | 6.[illegible] | 6.[illegible] | 5.[illegible] | [illegible] |
| 100.0 | 7.957 | 8.189 | 8.126 | 7.811 | 7.408 | 6.[illegible] | 6.[illegible] | 6.[illegible] | 6.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 200.0 | 9.[illegible]24 | 9.137 | 8.693 | 8.[illegible]61 | 7.468 | 6.[illegible] | 6.[illegible] | 6.171 | 5.[illegible] | 5.[illegible] | 5.[illegible] | [illegible] |
| 500.0 | 10.437 | 10.14[illegible] | 9.037 | 8.[illegible]66 | 7.285 | 6.[illegible] | 6.[illegible] | 5.[illegible] | 5.[illegible] | 4.[illegible] | 4.[illegible] | [illegible] |

**Table 9. Continued**

Z= 74

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | 1.026 | 1.033 | 1.042 | 1.052 | 1.064 | 1.079 | 1.096 | 1.117 | 1.140 | 1.168 | 1.183 | 1.199 |
| 2.5 | 2.012 | 2.044 | 2.092 | 2.147 | 2.211 | 2.282 | 2.359 | 2.445 | 2.534 | 2.625 | 2.671 | 2.716 |
| 5.0 | 2.961 | 2.983 | 3.027 | 3.083 | 3.208 | 3.319 | 3.539 | 3.664 | 3.759 | 3.851 | 3.894 | 3.928 |
| 10.0 | 3.979 | 4.000 | 4.056 | 4.311 | 4.463 | 4.607 | 4.765 | 4.783 | 4.653 | 4.859 | 4.869 | 4.863 |
| 25.0 | 5.237 | 5.498 | 5.641 | 5.912 | 5.912 | 5.946 | 5.926 | 5.868 | 5.787 | 5.693 | 5.644 | 5.594 |
| 50.0 | 6.286 | 6.530 | 6.742 | 6.823 | 6.783 | 6.668 | 6.516 | 6.351 | 6.185 | 6.024 | 5.947 | 5.873 |
| 100.0 | 7.349 | 7.428 | 7.735 | 7.617 | 7.373 | 7.216 | 6.845 | 6.642 | 6.381 | 6.179 | 6.089 | 5.997 |
| 200.0 | 8.414 | 8.601 | 8.852 | 8.796 | 7.664 | 7.273 | 6.933 | 6.639 | 6.385 | 6.163 | 6.062 | 5.968 |
| 500.0 | 9.826 | 9.926 | 9.977 | 9.832 | 7.661 | 7.156 | 6.712 | 6.344 | 6.04 | 5.797 | 5.695 | 5.613 |

Z= 82

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | .963 | .969 | .979 | .993 | 1.009 | 1.029 | 1.056 | 1.091 | 1.136 | 1.179 | 1.202 | 1.225 |
| 2.5 | 1.915 | 1.953 | 2.008 | 2.072 | 2.142 | 2.222 | 2.315 | 2.424 | 2.549 | 2.687 | 2.757 | 2.825 |
| 5.0 | 2.808 | 2.866 | 2.969 | 3.087 | 3.213 | 3.346 | 3.494 | 3.659 | 3.839 | 4.030 | 4.127 | 4.222 |
| 10.0 | 3.771 | 3.911 | 4.041 | 4.192 | 4.342 | 4.477 | 4.593 | 4.676 | 4.735 | 4.769 | 4.778 | 4.783 |
| 25.0 | 5.129 | 5.372 | 5.524 | 5.599 | 5.616 | 5.585 | 5.548 | 5.502 | 5.732 | 5.647 | 5.601 | 5.554 |
| 50.0 | 6.178 | 6.422 | 6.633 | 6.787 | 6.786 | 6.668 | 6.469 | 6.314 | 6.155 | 6.009 | 5.925 | 5.853 |
| 100.0 | 7.239 | 7.517 | 7.642 | 7.647 | 7.333 | 7.081 | 6.833 | 6.594 | 6.376 | 6.178 | 6.085 | 5.992 |
| 200.0 | 8.303 | 8.565 | 8.447 | 8.177 | 7.659 | 7.293 | 6.962 | 6.675 | 6.424 | 6.213 | 6.111 | 6.017 |
| 500.0 | 9.725 | 9.778 | 9.773 | 9.555 | 7.712 | 7.213 | 6.796 | 6.443 | 6.169 | 5.923 | 5.822 | 5.726 |

Z= 92

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | .895 | .899 | .914 | .941 | .917 | .916 | .937 | .965 | .965 | .984 | .994 | 1.005 |
| 2.5 | 1.820 | 1.854 | 1.904 | 1.941 | 1.992 | 2.052 | 2.119 | 2.192 | 2.271 | 2.354 | 2.398 | 2.433 |
| 5.0 | 2.694 | 2.767 | 2.859 | 2.944 | 3.049 | 3.160 | 3.274 | 3.387 | 3.494 | 3.591 | 3.637 | 3.679 |
| 10.0 | 3.659 | 3.778 | 3.913 | 4.059 | 4.216 | 4.343 | 4.461 | 4.553 | 4.621 | 4.665 | 4.679 | 4.688 |
| 25.0 | 5.008 | 5.197 | 5.349 | 5.508 | 5.617 | 5.715 | 5.757 | 5.725 | 5.666 | 5.591 | 5.547 | 5.517 |
| 50.0 | 6.156 | 6.394 | 6.576 | 6.617 | 6.619 | 6.515 | 6.412 | 6.268 | 6.118 | 5.969 | 5.897 | 5.827 |
| 100.0 | 7.115 | 7.396 | 7.586 | 7.47 | 7.279 | 7.144 | 6.815 | 6.576 | 6.364 | 6.169 | 6.078 | 5.992 |
| 200.0 | 8.079 | 8.403 | 8.576 | 8.340 | 7.648 | 7.311 | 6.981 | 6.699 | 6.451 | 6.231 | 6.127 | 6.036 |
| 500.0 | 9.591 | 9.681 | 9.763 | 8.565 | 7.773 | 7.268 | 6.867 | 6.552 | 6.275 | 6.039 | 5.926 | 5.841 |

**Table 9. Continued**

Z= 54

| K/T1 / T1 | 0.000 | 0.100 | 0.200 | 0.300 | 0.400 | 0.500 | 0.600 | 0.700 | 0.800 | 0.900 | 0.950 | 0.990 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.0 | 1.232 | 1.246 | 1.261 | 1.279 | 1.301 | 1.326 | 1.355 | 1.388 | 1.426 | 1.469 | 1.492 | 1.511 |
| 2.5 | 2.276 | 2.331 | 2.391 | 2.462 | 2.542 | 2.635 | 2.735 | 2.844 | 2.964 | 3.094 | 3.162 | 3.217 |
| 5.0 | 3.245 | 3.352 | 3.413 | 3.535 | 3.666 | 3.807 | 3.922 | 4.033 | 4.139 | 4.239 | 4.289 | 4.368 |
| 10.0 | 4.241 | 4.348 | 4.511 | 4.677 | 4.837 | 4.993 | 5.141 | 5.283 | 5.414 | 5.531 | 5.591 | 5.688 |
| 25.0 | 5.571 | 5.735 | 5.936 | 6.149 | 6.215 | 6.295 | 6.143 | 6.049 | 5.892 | 5.754 | 5.701 | 5.711 |
| 50.0 | 6.625 | 6.881 | 7.166 | 7.391 | 6.909 | 6.822 | 6.631 | 6.437 | 6.227 | 6.074 | 5.991 | 5.911 |
| 100.0 | 7.687 | 7.949 | 7.976 | 7.795 | 7.443 | 7.125 | 6.931 | 6.566 | 6.332 | 6.124 | 6.024 | 5.943 |
| 200.0 | 8.753 | 8.937 | 8.625 | 8.093 | 7.536 | 7.142 | 6.755 | 6.434 | 6.162 | 5.934 | 5.835 | 5.743 |
| 500.0 | 10.166 | 10.110 | 9.063 | 8.176 | 7.497 | 6.987 | 6.379 | 5.947 | 5.567 | 5.3 | 5.191 | 5.1 |

Table 10. Ratio of approximated to exact numerical bremsstrahlung data (from Table 8 and Table 9).

Z= 13

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | [illegible] | [illegible] | 0.647 | 0.655 | 0.669 | [illegible] | 0.7 4 | [illegible] | 0.723 | [illegible] | 0.743 | [illegible] | [illegible] | [illegible] |
| 2.5 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 10.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 25.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 50.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 100.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 200.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 500.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Z= 26

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 2.5 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 10.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 25.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 50.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 100.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 200.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 500.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

Z= 42

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 2.5 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 5.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 10.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 25.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 50.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 100.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 200.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| 500.0 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |

**Table 10. Continued**

Z= 54

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | 1.264 | 1.135 | 1.113 | 1.050 | 0.996 | 0.948 | 0.904 | 0.866 | 0.844 | 0.825 | 0.811 | 0.7 |
| 2.5 | 1.197 | 1.143 | 1.102 | 1.074 | 1.055 | 1.042 | 1.033 | 1.028 | 1.021 | 1.017 | 1.014 | 1.0 |
| 5.0 | 1.153 | 1.121 | 1.098 | 1.081 | 1.068 | 1.058 | 1.051 | 1.046 | 1.041 | 1.037 | 1.034 | 1.0 |
| 10.0 | 0.953 | 0.939 | 0.952 | 0.977 | 1.004 | 1.028 | 1.046 | 1.060 | 1.071 | 1.080 | 1.087 | 1.0 |
| 25.0 | 0.877 | 0.889 | 0.933 | 0.979 | 1.012 | 1.036 | 1.054 | 1.068 | 1.079 | 1.088 | 1.095 | 1.1 |
| 50.0 | 0.847 | 0.882 | 0.949 | 1.004 | 1.052 | 1.087 | 1.113 | 1.133 | 1.149 | 1.161 | 1.171 | 1.1 |
| 100.0 | 0.819 | 0.896 | 0.934 | 1.042 | 1.100 | 1.140 | 1.170 | 1.192 | 1.209 | 1.222 | 1.232 | 1.2 |
| 200.0 | 0.826 | 0.946 | 1.043 | 1.112 | 1.169 | 1.210 | 1.240 | 1.263 | 1.281 | 1.295 | 1.306 | 1.3 |
| 500.0 | 0.895 | 1.050 | 1.136 | 1.205 | 1.257 | 1.299 | 1.330 | 1.354 | 1.374 | 1.389 | 1.402 | 1.4 |

Z= 74

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | 1.671 | 1.554 | 1.441 | 1.337 | 1.247 | 1.169 | 1.102 | 1.044 | 0.994 | 0.953 | 0.918 | 0.8 |
| 2.5 | 1.511 | 1.422 | 1.348 | 1.288 | 1.239 | 1.199 | 1.167 | 1.141 | 1.119 | 1.101 | 1.086 | 1.0 |
| 5.0 | 1.242 | 1.186 | 1.140 | 1.103 | 1.116 | 1.093 | 1.077 | 1.064 | 1.054 | 1.046 | 1.039 | 1.0 |
| 10.0 | 1.075 | 1.047 | 1.046 | 1.058 | 1.075 | 1.091 | 1.102 | 1.112 | 1.120 | 1.126 | 1.131 | 1.1 |
| 25.0 | 0.941 | 0.944 | 0.979 | 1.019 | 1.053 | 1.075 | 1.091 | 1.104 | 1.114 | 1.122 | 1.128 | 1.1 |
| 50.0 | 0.870 | 0.909 | 0.961 | 1.013 | 1.047 | 1.071 | 1.088 | 1.101 | 1.111 | 1.119 | 1.125 | 1.1 |
| 100.0 | 0.827 | 0.888 | 0.969 | 1.035 | 1.079 | 1.112 | 1.137 | 1.156 | 1.171 | 1.183 | 1.192 | 1.2 |
| 200.0 | 0.813 | 0.918 | 1.031 | 1.104 | 1.164 | 1.210 | 1.245 | 1.272 | 1.293 | 1.309 | 1.322 | 1.3 |
| 500.0 | 0.861 | 1.012 | 1.097 | 1.158 | 1.222 | 1.271 | 1.309 | 1.339 | 1.363 | 1.382 | 1.397 | 1.4 |

Z= 82

| K/T1 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | | | | | | | | | | | | |
| 1.0 | 1.603 | 1.664 | 1.534 | 1.435 | 1.347 | 1.270 | 1.202 | 1.144 | 1.094 | 1.051 | 1.014 | 0.9 |
| 2.5 | 1.615 | 1.514 | 1.427 | 1.357 | 1.299 | 1.249 | 1.207 | 1.171 | 1.140 | 1.114 | 1.092 | 1.0 |
| 5.0 | 1.338 | 1.245 | 1.262 | 1.175 | 1.158 | 1.147 | 1.142 | 1.139 | 1.139 | 1.142 | 1.148 | 1.1 |
| 10.0 | 1.123 | 1.091 | 1.085 | 1.093 | 1.106 | 1.119 | 1.128 | 1.135 | 1.139 | 1.141 | 1.141 | 1.1 |
| 25.0 | 0.967 | 0.966 | 0.999 | 1.033 | 1.061 | 1.083 | 1.101 | 1.115 | 1.126 | 1.135 | 1.142 | 1.1 |
| 50.0 | 0.883 | 0.929 | 0.967 | 1.007 | 1.041 | 1.069 | 1.091 | 1.108 | 1.122 | 1.133 | 1.142 | 1.1 |
| 100.0 | 0.825 | 0.896 | 0.965 | 1.019 | 1.062 | 1.096 | 1.122 | 1.142 | 1.158 | 1.170 | 1.180 | 1.1 |
| 200.0 | 0.810 | 0.911 | 1.011 | 1.083 | 1.138 | 1.182 | 1.216 | 1.243 | 1.264 | 1.281 | 1.294 | 1.3 |
| 500.0 | 0.853 | 0.999 | 1.086 | 1.161 | 1.220 | 1.268 | 1.305 | 1.335 | 1.359 | 1.378 | 1.393 | 1.4 |

Table 10. Continued

| Z= 92 | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| K/T1 | 0.000 | 0.101 | 0.201 | 0.302 | 0.402 | 0.503 | 0.603 | 0.704 | 0.804 | 0.905 | 1.005 | 1.106 |
| T0 | | | | | | | | | | | | |
| 1.0 | 1.938 | 1.774 | 1.617 | 1.479 | 1.357 | 1.256 | 1.168 | 1.093 | 1.032 | 0.982 | 1.374 | 0.987 |
| 2.5 | 1.723 | 1.658 | 1.508 | 1.427 | 1.361 | 1.307 | 1.264 | 1.231 | 1.205 | 1.189 | 1.184 | 1.177 |
| 5.0 | 1.381 | 1.311 | 1.263 | 1.233 | 1.212 | 1.198 | 1.188 | 1.181 | 1.174 | 1.163 | 1.157 | 1.149 |
| 10.0 | 1.185 | 1.146 | 1.134 | 1.138 | 1.147 | 1.156 | 1.162 | 1.163 | 1.156 | 1.142 | 1.132 | 1.12 |
| 25.0 | 1.096 | 0.995 | 1.025 | 1.062 | 1.095 | 1.115 | 1.121 | 1.118 | 1.106 | 1.087 | 1.076 | 1.063 |
| 50.0 | 0.899 | 1.021 | 0.975 | 1.024 | 1.057 | 1.075 | 1.082 | 1.081 | 1.071 | 1.058 | 1.043 | 0.979 |
| 100.0 | 0.829 | 0.826 | 0.961 | 1.013 | 1.043 | 1.061 | 1.070 | 1.075 | 1.079 | 1.082 | 1.091 | 1.09 |
| 200.0 | 0.839 | 0.954 | 0.991 | 1.042 | 1.077 | 1.105 | 1.128 | 1.150 | 1.17 | 1.191 | 1.208 | 1.213 |
| 500.0 | 0.844 | 0.984 | 1.069 | 1.123 | 1.173 | 1.214 | 1.241 | 1.255 | 1.268 | 1.277 | 1.283 | 1.287 |

Table 11. Comparisons for the ionic data.

Al (Z = 13)

| $T_1$ | $Z_i$ / $k/T_1$ | 9 | | | 11 | | |
|---|---|---|---|---|---|---|---|
| | | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ |
| 1 | 0.95 | 4.858 | 4.225 | .869 | 5.236 | 5.057 | .966 |
| | 0.8 | 4.947 | 4.246 | .858 | 5.367 | 5.220 | .973 |
| | 0.6 | 5.119 | 4.32 | .844 | 5.603 | 5.524 | .986 |
| 5 | 0.95 | 5.659 | 5.549 | .980 | 5.741 | 5.757 | 1.003 |
| | 0.8 | | | | | | |
| | 0.6 | 6.205 | 5.95 | .959 | | | |
| 10 | 0.95 | | | | 5.625 | 5.638 | 1.002 |
| | 0.8 | | | | | | |
| | 0.6 | | | | | | |
| 50 | 0.95 | | | | 4.135 | 4.153 | 1.004 |
| | 0.8 | | | | | | |
| | 0.6 | | | | | | |

Table 11. Continued

Fe ($Z$ = 26)

| $T_1$ | $Z_i$ / $k/T_1$ | 8 | | | 16 | | | 22 | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\sigma_{exact}$ | $\sigma_{est.}$ | $\frac{\sigma_{est.}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est.}$ | $\frac{\sigma_{est.}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est.}}{\sigma_{exact}}$ |
| 1 | 0.95 | | | | 4.958 | 4.728 | .954 | | | |
| | 0.8 | | | | 5.035 | 4.900 | .973 | | | |
| | 0.6 | | | | 5.180 | 4.940 | .954 | | | |
| 5 | 0.95 | 4.985 | 4.947 | .992 | 5.095 | 5.227 | 1.026 | 5.581 | 5.740 | 1.028 |
| | 0.8 | 5.037 | 4.936 | .980 | 5.152 | 5.295 | 1.028 | 5.744 | 5.951 | 1.036 |
| | 0.6 | 5.103 | 4.838 | .948 | 5.254 | 5.351 | 1.084 | 6.032 | 6.288 | 1.042 |
| 10 | 0.95 | 5.454 | 5.537 | 1.015 | 5.526 | 5.693 | 1.030 | 5.759 | 5.980 | 1.038 |
| | 0.8 | 5.599 | 5.655 | 1.010 | 5.676 | 5.863 | 1.033 | 5.976 | 6.243 | 1.045 |
| | 0.6 | 5.820 | 5.752 | .988 | | | | 6.348 | 6.660 | 1.049 |
| 50 | 0.95 | 5.018 | 5.678 | 1.132 | | | | 5.076 | 5.142 | 1.013 |
| | 0.8 | | | | | | | 5.457 | 5.522 | 1.012 |
| | 0.6 | | | | | | | 6.122 | 6.202 | 1.013 |

Table 11. Continued

Mo (Z = 42)

| $T_1$ | $Z_i$ / $k/T_1$ | 24 $\sigma_{exact}$ | 24 $\sigma_{est}$ | 24 $\frac{\sigma_{est}}{\sigma_{exact}}$ | 32 $\sigma_{exact}$ | 32 $\sigma_{est}$ | 32 $\frac{\sigma_{est}}{\sigma_{exact}}$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.95 | 3.4 | 2.61 | .767 | 4.0 | 3.76 | .940 |
| | 0.8 | 3.35 | 2.552 | .761 | 4.0 | 3.75 | .938 |
| | 0.6 | 3.3 | 2.501 | .758 | 4.0 | 3.76 | .940 |
| 5 | 0.95 | 4.8 | 4.82 | 1.004 | 5.0 | 5.28 | 1.056 |
| | 0.8 | 4.75 | 4.82 | 1.015 | 5.0 | 5.40 | 1.08 |
| | 0.6 | 4.7 | 4.75 | 1.011 | 5.1 | 5.48 | 1.075 |
| 10 | 0.95 | 5.1 | 5.46 | 1.071 | 5.3 | 5.72 | 1.079 |
| | 0.8 | 5.2 | 5.56 | 1.069 | 5.5 | 5.90 | 1.073 |
| | 0.6 | 5.3 | 5.66 | 1.068 | 5.6 | 6.17 | 1.102 |
| 50 | 0.95 | 5.3 | 6.03 | 1.138 | 5.4 | 6.11 | 1.131 |
| | 0.8 | 5.8 | 6.32 | 1.089 | 5.9 | 6.42 | 1.088 |
| | 0.6 | 6.3 | 6.78 | 1.076 | 6.2 | 6.94 | 1.119 |
| 100 | 0.95 | 4.8 | 5.55 | 1.156 | 4.8 | 5.08 | 1.058 |
| | 0.8 | 5.1 | 5.89 | 1.155 | 5.1 | 5.47 | 1.073 |
| | 0.6 | 6.0 | 6.49 | 1.082 | 6.0 | 6.15 | 1.025 |

Table 11. Continued

W (Z = 74)

| $T_1$ | $Z_i$ / $k/T_1$ | 38 | | | 56 | | | 64 | | | 74 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ | $\sigma_{exact}$ | $\sigma_{est}$ | $\frac{\sigma_{est}}{\sigma_{exact}}$ |
| 1 | 0.95 | | | | | | | | | | | | |
| | 0.8 | | | | 3 664 | 5.566 | 1 759 | | | | | | |
| | 0.6 | | | | | | | | | | | | |
| 5 | 0.95 | 3.831 | 4.215 | 1.100 | | | | 4.989 | 5.438 | 1.090 | 5.910 | 6.093 | 1.031 |
| | 0.8 | 3.761 | 4.127 | 1.097 | | | | 5.051 | 5.517 | 1.092 | 6.008 | 6.262 | 1.042 |
| | 0.6 | 3.664 | 3.960 | 1.081 | | | | 5.173 | 5.549 | 1.073 | 6.191 | 6.400 | 1.034 |
| 10 | 0.95 | 4.604 | 5.055 | 1.098 | | | | 5.329 | 5.781 | 1.085 | 6.092 | 6.170 | 1.013 |
| | 0.8 | 4.568 | 5.079 | 1.119 | | | | 5.408 | 6.006 | 1.111 | 6.218 | 6.502 | 1.046 |
| | 0.6 | 4.512 | 5.00 | 1.108 | | | | 5.563 | 6.116 | 1.099 | 6.459 | 6.714 | 1.039 |
| 50 | 0.95 | | | | 5.800 | 6.092 | 1.050 | 5.891 | 6.164 | 1.046 | | | |
| | 0.8 | | | | 6.015 | 6.385 | 1.062 | 6.120 | 6.483 | 1.059 | | | |
| | 0.6 | | | | 6.359 | 6.487 | 1.077 | 6.499 | 7.010 | 1.079 | | | |
| 100 | 0.95 | | | | 5.502 | 6.170 | 1.121 | | | | | | |
| | 0.8 | | | | 5.862 | 6.495 | 1.108 | | | | | | |
| | 0.6 | | | | 6.427 | 7.033 | 1.094 | | | | | | |

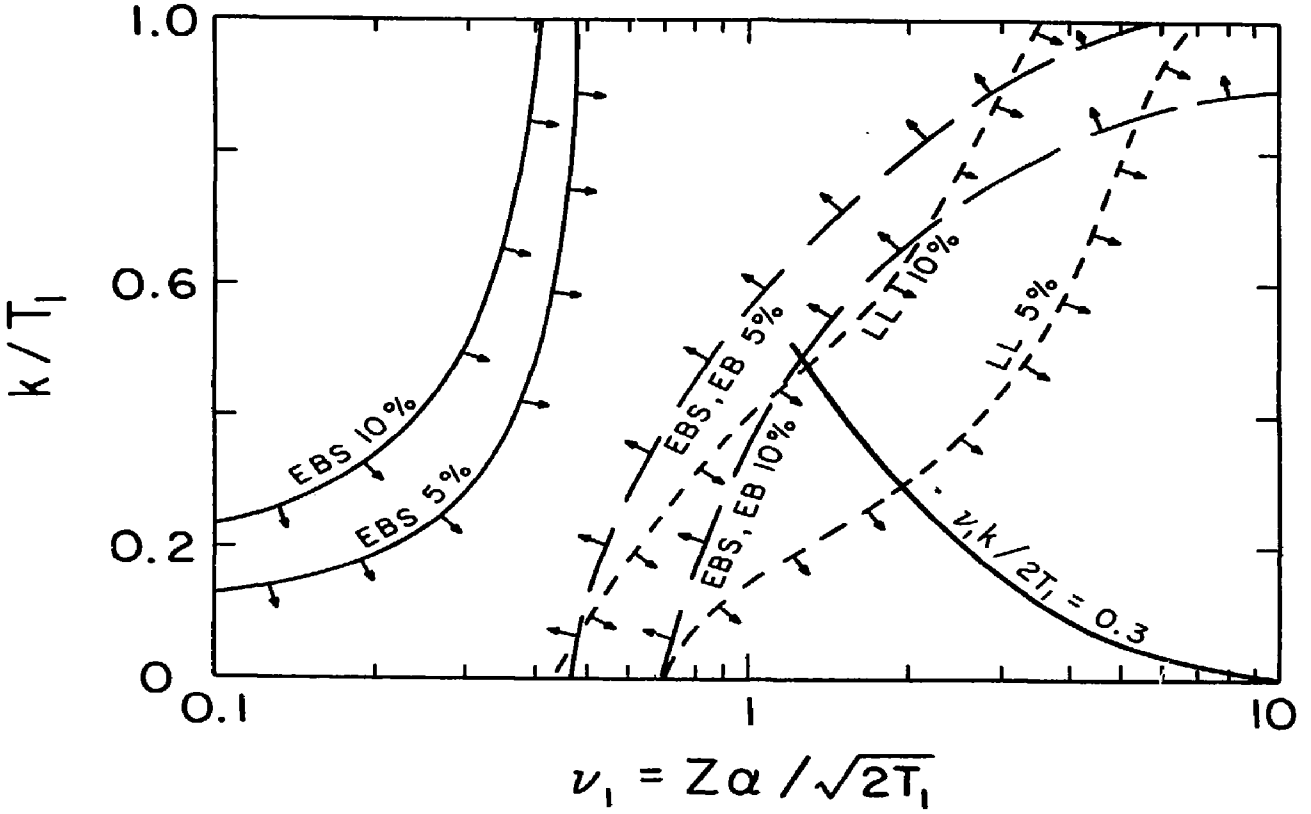
1.0
0.6
0.2
0
k / T1
0.1
1
10
ν1 = Zα / √2T1
EBS 10%
EBS 5%
EBS, EB 5%
EBS, EB 10%
LL T 10%
LL 5%
ν1 k / 2T1 = 0.3

Figure 1

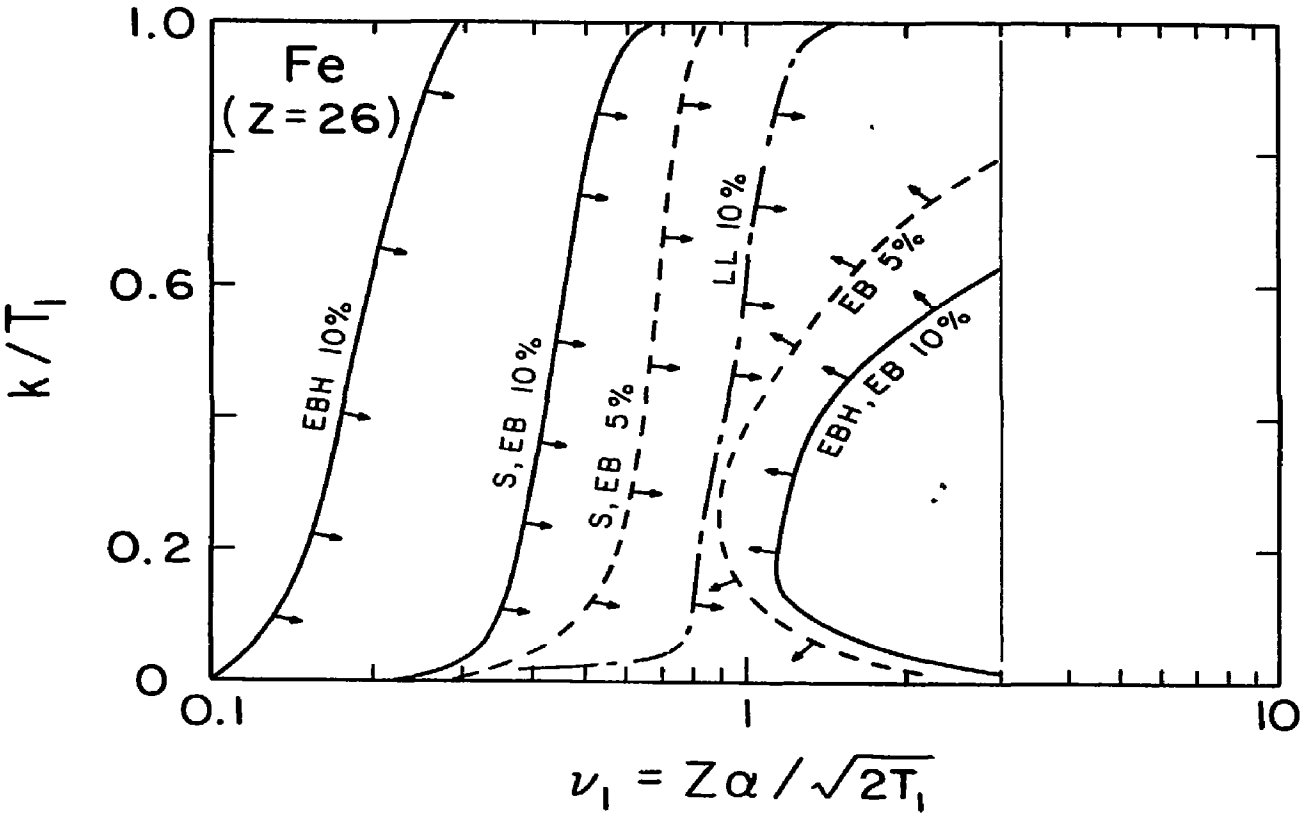
Fe
(Z=26)
1.0
0.6
0.2
0
0.1
1
10
k/T1
EBH 10%
S,EB 10%
S,EB 5%
LL 10%
EB 5%
EBH,EB 10%
ν1 = Zα/√2T1

Figure 2

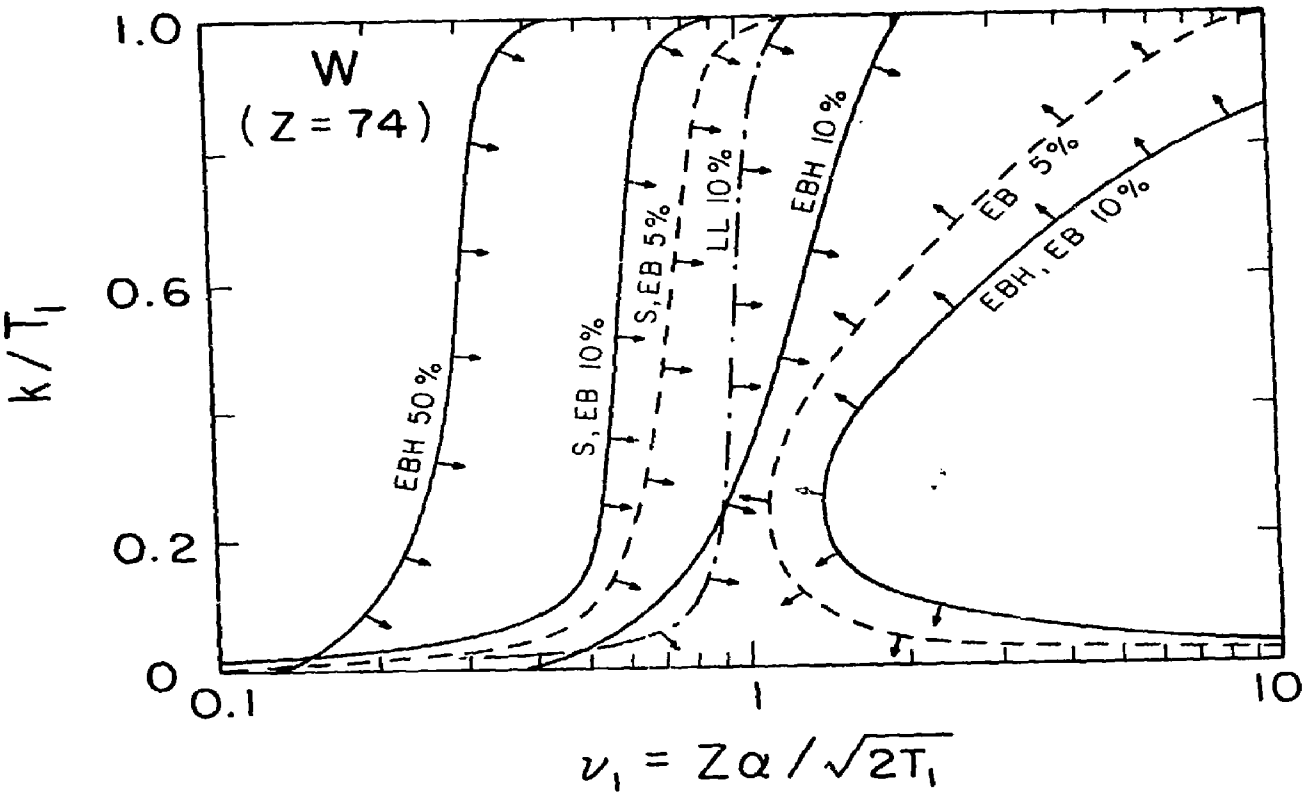
W
(Z = 74)
1.0
0.6
0.2
0
0.1
1
10
k/T1
ν1 = Zα/√2T1
EBH 50 %
S, EB 10%
S, EB 5%
LL 10%
EBH 10%
EB 5%
EBH, EB 10%

Figure 3

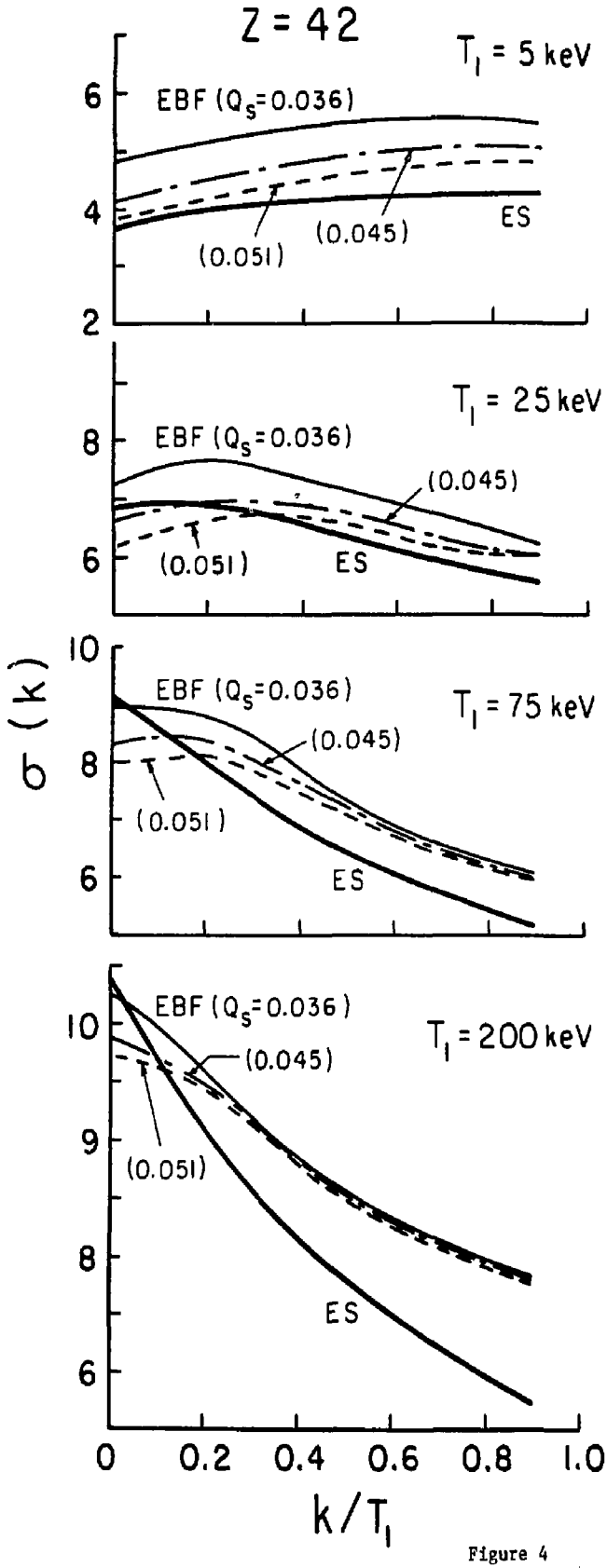
Z = 42
$T_I$ = 5 keV
EBF ($Q_s$=0.036)
(0.051)
(0.045)
ES
$T_I$ = 25 keV
EBF ($Q_s$=0.036)
(0.045)
(0.051)
ES
$T_I$ = 75 keV
EBF ($Q_s$=0.036)
(0.045)
(0.051)
ES
$T_I$ = 200 keV
EBF ($Q_s$=0.036)
(0.045)
(0.051)
ES
b (k)
k/$T_I$

Figure 4

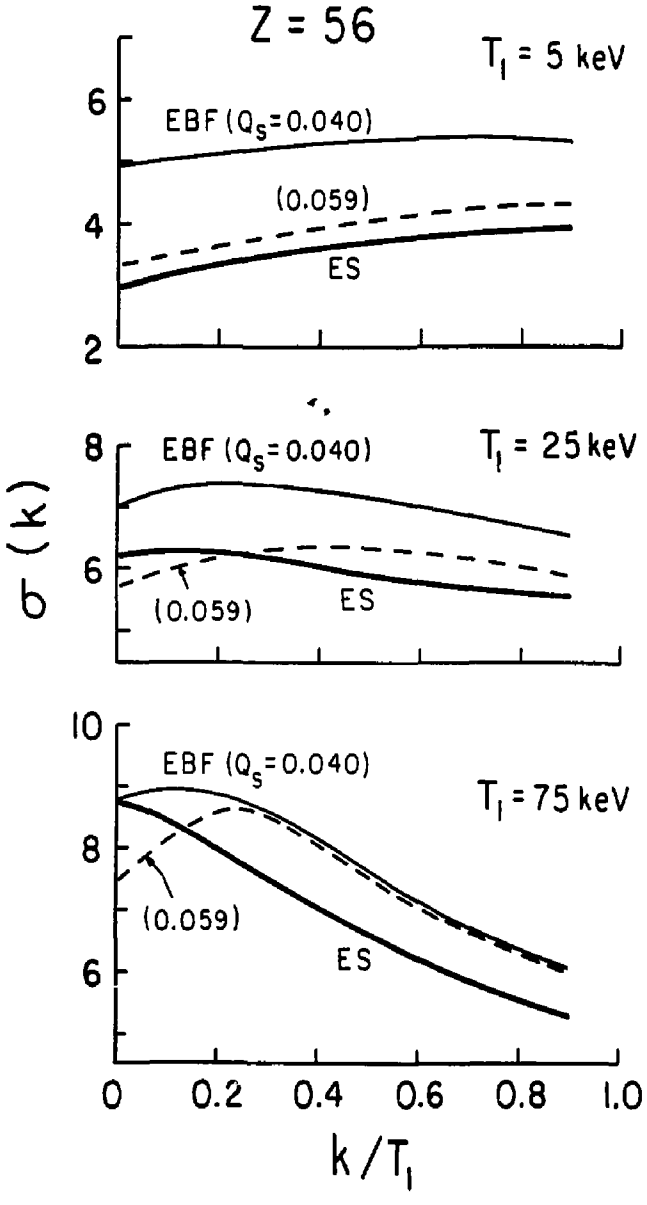
Z = 56
$T_i$ = 5 keV
EBF ($Q_s$=0.040)
(0.059)
ES
$T_i$ = 25 keV
$T_i$ = 75 keV
b (k)
k/$T_i$
0
0.2
0.4
0.6
0.8
1.0
2
4
6
8
10

Figure 5

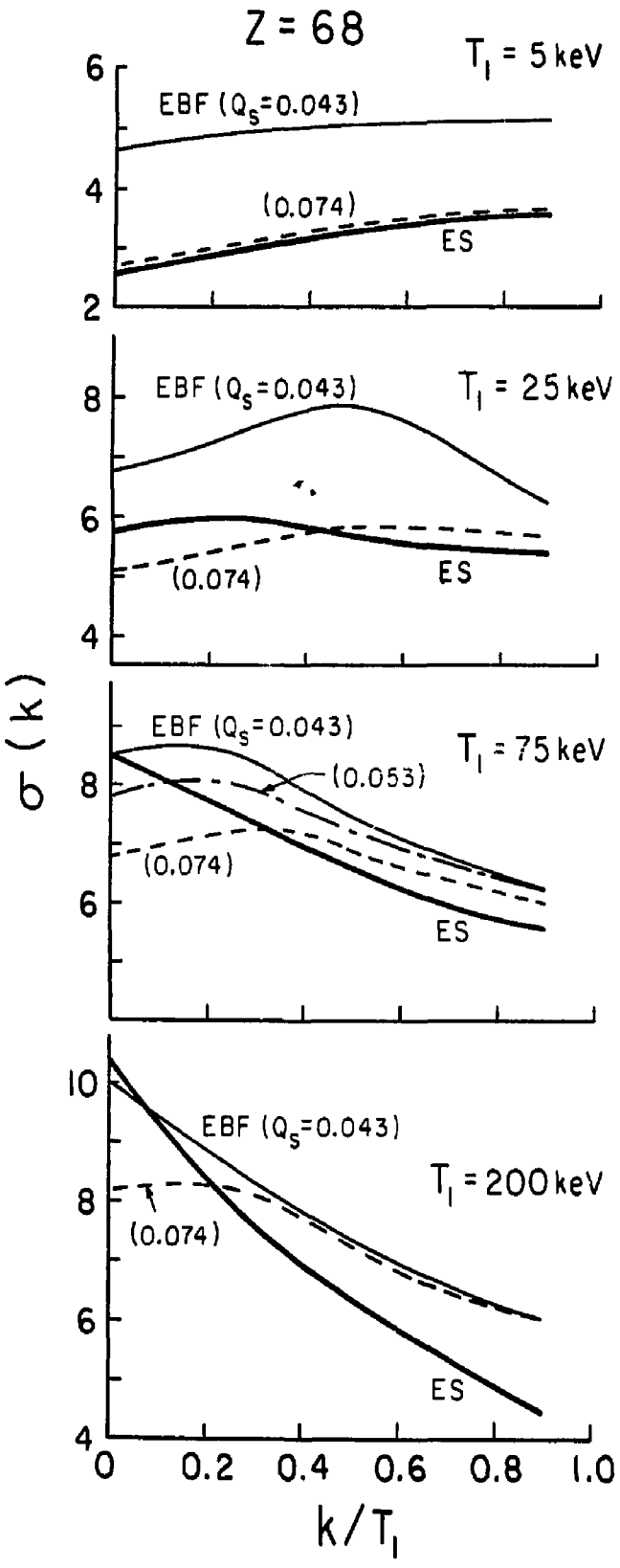
Z = 68
T_I = 5 keV
EBF (Q_S=0.043)
(0.074)
ES
T_I = 25 keV
T_I = 75 keV
(0.053)
T_I = 200 keV
σ (k)
k/T_I
0
0.2
0.4
0.6
0.8
1.0

Figure 6

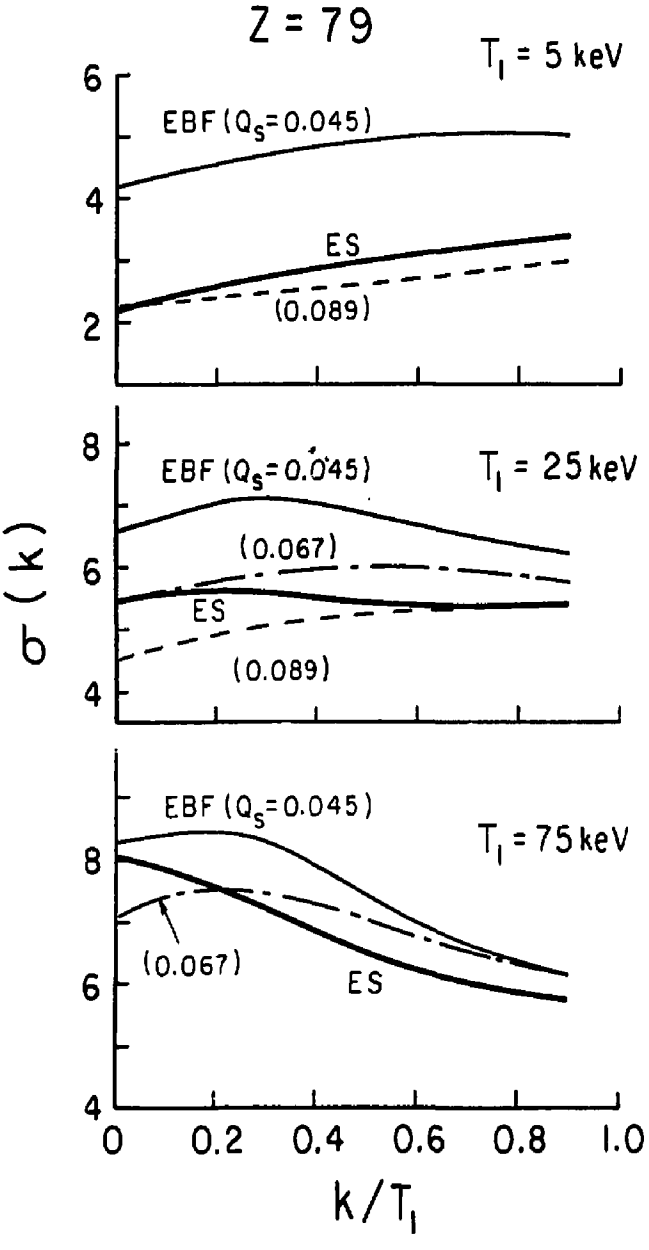
Z = 79
$T_I$ = 5 keV
EBF ($Q_S$=0.045)
ES
(0.089)
$T_I$ = 25 keV
EBF ($Q_S$=0.045)
(0.067)
ES
(0.089)
$T_I$ = 75 keV
EBF ($Q_S$=0.045)
(0.067)
ES
b (k)
k/$T_I$

Figure 7

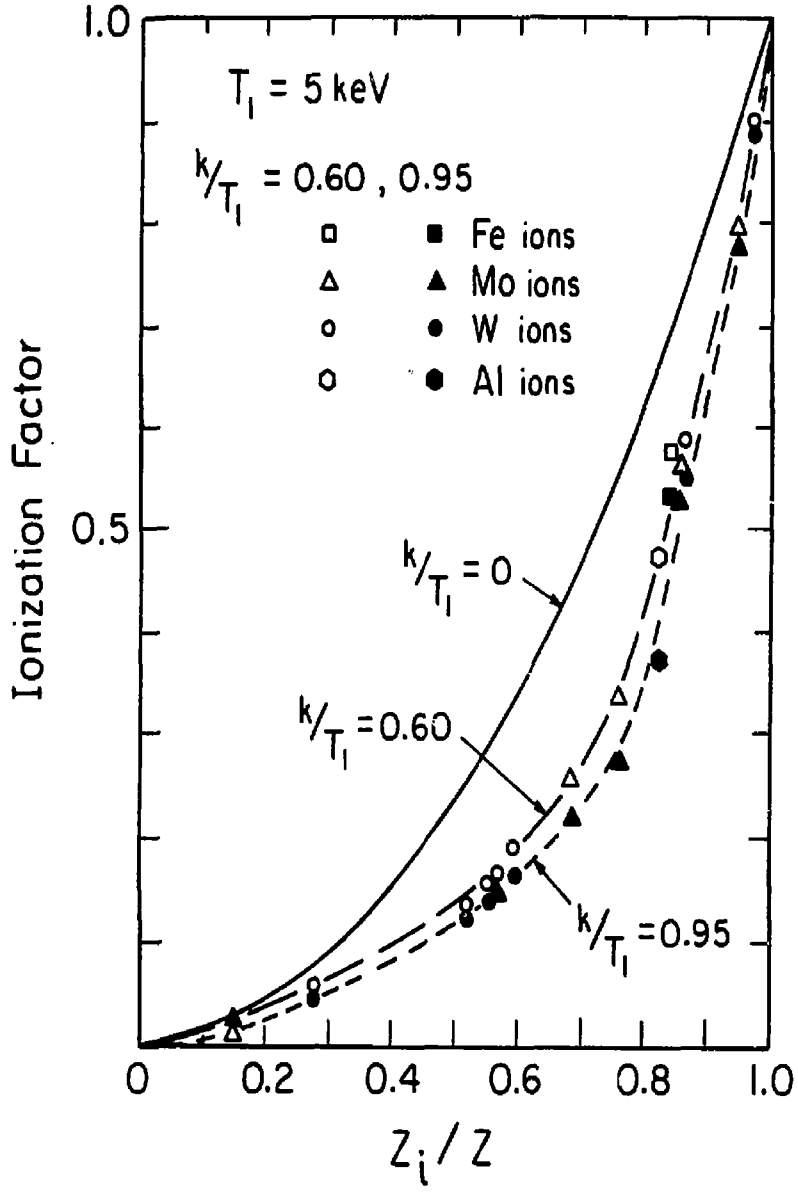
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
$T_i$ = 5 keV
$k/T_i$ = 0.60 , 0.95
Fe ions
Mo ions
W ions
Al ions
$k/T_i$ = 0
$k/T_i$ = 0.60
$k/T_i$ = 0.95
Ionization Factor
$Z_i / Z$

Figure 8

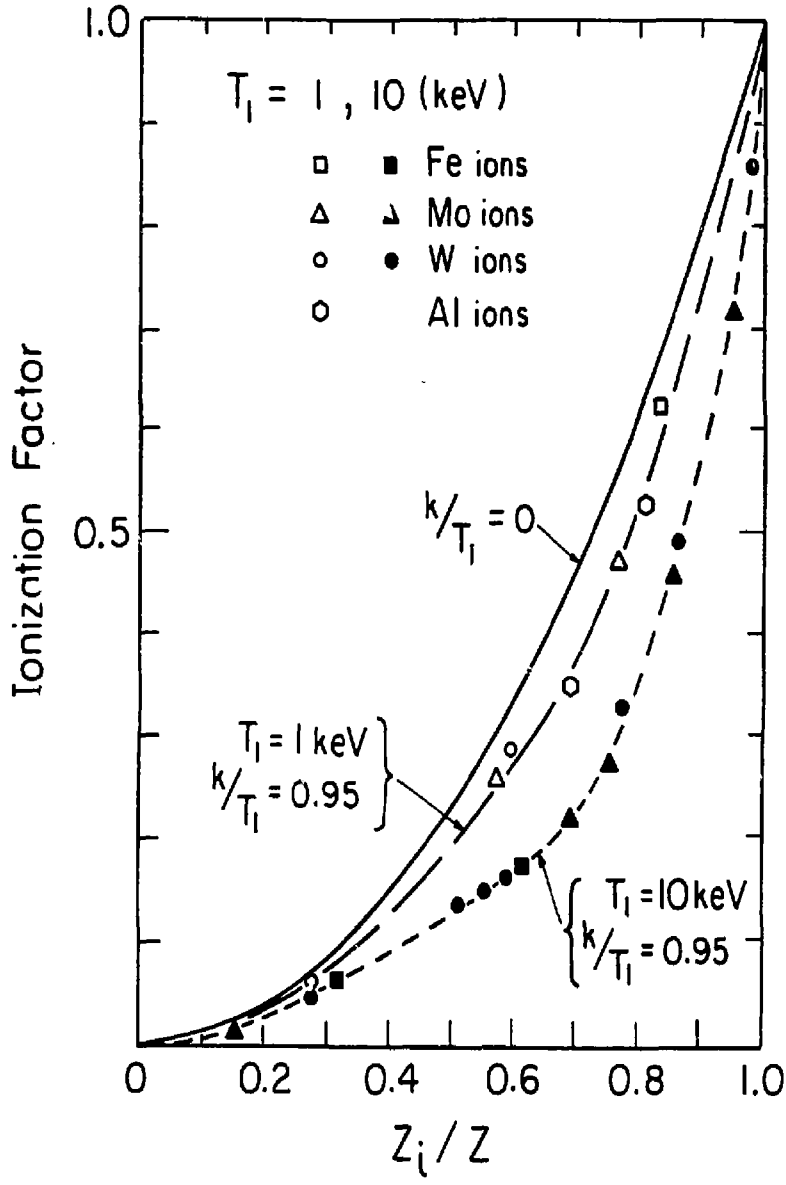
1.0
0.5
0
0.2
0.4
0.6
0.8
1.0
Ionization Factor
$Z_i / Z$
$T_I$ = 1 , 10 (keV)
Fe ions
Mo ions
W ions
Al ions
$k/T_I = 0$
$T_I$ = 1 keV
$k/T_I$ = 0.95
$T_I$ = 10 keV
$k/T_I$ = 0.95

Figure 9